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# ELECTRICITY

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# FOUNDATIONS OF TECHNICAL ELECTRICITY

BY
E. MALLETT
D.Sc., M.Inst.C.E., M.I.E.E., F.Inst.P., etc.
FELLOW OF UNIVERSITY COLLEGE, LONDON
PRINCIPAL AND HEAD OF THE ELECTRICAL ENGINEERING
DEPARTMENT, WOOLWICH POLYTECHNIC

AND

T. B. VINYCOMB
M.C., M.A., F.Inst.P.
HEAD OF THE PHYSICS DEPARTMENT
WOOLWICH POLYTECHNIC

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# PREFACE

The appearance of yet another textbook on Elementary Electricity and Magnetism may seem to call for some justification. It is not that all the others are bad—some are very good in their own particular way—but their way is different from that of the present book. Too much the existing books have followed the traditional method. In the beginning there is on the one hand an ebonite rod and a piece of flannel, and on the other hand a permanent magnet, then there is the inverse square law and the earth's magnetic field and the tangent galvanometer.

We have started with an electric current, have gone on to electric charges and fields, proceeded to the magnetic effects of currents and magnetic fields, and finished with the chemical effects. Throughout, since the alternating current is fast becoming used far more than the direct current, we have studied alternating currents as well as direct currents. We think there is no difficulty in doing this; the advantages are obvious. The ebonite rod and the permanent magnet are relegated to their proper place. We have avoided the idea of a charge of magnetism, a stumbling block to many. The basis of electro-magnetism is experimental; the field produced by a current and the e.m.f. produced by the relative motion of a conductor and a field are the starting points.

In this way that part of the subject which is essential in all applications in Electrical Engineering and in Telecommunications is reached much earlier than by the old method.

This is of the utmost importance to evening students proceeding to National Certificates and City and Guilds Examinations. It can in fact be stated that the book contains the minimum foundations of the pure science of the subject required for the building of a proper understanding of Electrical Engineering and Telecommunications. We think that the method should also be used by all other students, whatever their objective. If a more academic study is required, it could very well follow the treatment in the book.

It would be lacking in courtesy not to mention the names of Dr. F. T. Chapman, H.M.I., and Professor C. L. Fortescue, of the City and Guilds (Eng.) College, who have on many occasions to one or other of the authors expressed their dissatisfaction with the old method and envisaged something of the method now presented. How far our ideas are based on these discussions it is impossible to say; we feel they should in justice be mentioned in connection with any merit that may be found in the book; but also in justice we feel that they must not in any way be held responsible for any deficiencies.

We also wish to thank the Technical Editor of Sir Isaac Pitman & Sons, Ltd., and his assistant, for their unfailing courtesy and great care and skill in correcting the proofs of the book, and for making many helpful suggestions.

E. M.
T. B. V.

# NOTE TO STUDENTS

THOSE students who have no previous knowledge of the subject might well omit certain parts of the book at a first reading. It is suggested that they should read—

Chap. I, paras. 1, 2, 3, 4, 6 and 7, omitting the paragraph on p. 10, beginning "The energy" and ending "$= rv^2$," 8, 9, 10, 11, 13, 15, 16 from p. 28, middle of page, beginning at "These have four terminals"—to end of paragraph.

Chap. II, paras. 1, 2, 7, 8, to p. 68 ending at . . . "charge would move."

Chap. III, paras, 1, 2 to p. 84, ending at ". . . of the lines of flux." 3, 5, down to ". . . lines per cm$^2$." p. 95 and on p. 95 first para. only, 7, 8.

Chap. IV, paras. 1, 2 to p. 118 ending at "capital lambda."

Chap. V.

Then read from the beginning straight through.

# TABLE OF QUANTITIES AND SYMBOLS

| Term | Symbol | Unit | Abbreviations | |
|---|---|---|---|---|
| | | | For Unit | For Term |
| Work | $A$ | joule | J. | |
| Energy | $W$ | | | |
| Power | $P$ | watt | W. | |
| Number of revolutions in unit of time | $n$ | | | |
| Temperature | $\tau$ | | | |
| Frequency | $n$ or $f$ | | | |
| Angular velocity | $\omega$ | | | |
| Phase difference | $\phi$ | | | |
| Power factor | $\cos\phi$* | | | p.f. |
| Quantity of electricity | $Q$ | coulomb<br>ampere-hour | C.<br>Ah. | |
| Electric force | $\varepsilon$ | | | |
| Electric flux | $\Psi$ | | | |
| Electric flux density | $D$ | | | |
| Potential difference (electric) | $V$ | volt | V. | p.d. |
| Electromotive force | $E$ | | | e.m.f. |
| Current | $I$ | ampere | A. or amp. | |
| Magnetic field strength | $H$ | gauss or line per cm.$^2$ | | |
| Magnetic flux | $\Phi$ | line | | |
| Magnetic flux density | $B$ | line per cm.$^2$ | | |
| Magnetic motive force | $F$ | | | m.m.f. |
| Reluctance | $S$ | | | |
| Permeability | $\mu$ | | | |
| Intensity of Magnetization | $J$ | | | |
| Resistance | $R$ | ohm | Ω. | |
| Resistivity | $\rho$ | | | |
| Conductivity | $\sigma$ | | | |
| Capacitance | $C$ | farad | F. | |
| Dielectric constant | $\kappa$ | | | |
| Self-inductance | $L$ | henry | H. | |
| Mutual inductance | $M$ | | | |
| Reactance | $X$ | | | |
| Impedance ($R + jX$) | $Z$ | | | |
| Root mean square | | | | r.m.s. |

---

* For sinusoidal quantities only.

## TABLE OF QUANTITIES AND SYMBOLS (*contd.*)

| Prefix | Meaning | Abbreviation |
|---|---|---|
| micro-micro or pica . | $10^{-12}$ | $\mu\mu$ |
| micro . . . | one millionth | $\mu$ |
| milli . . . | one thousandth | m. |
| centi . . . | one hundredth | c. |
| kilo . . . | one thousand | k. |
| mega . . . | one million | M. |
| e.g. micromicrofarad . . . |  | $\mu\mu$F. |
| microfarad . . . . |  | $\mu$F. |
| milliampere . . . . |  | mA. |
| kilowatt . . . . |  | kW. |
| megohm . . . . |  | MΩ. |

# CONTENTS

PAGE

PREFACE . . . . . . . . . . V

. . . VII

## CHAPTER I

ELECTRIC CURRENTS . . . . . . . . 1

Conductors and insulators—Electrolysis—Supply—Quantity current—Electrochemical equivalent—Potential difference—Resistance and Ohm's law—Hot-wire ammeter—Alternating currents and potential differences—Series and parallel connection—Resistivity and conductivity—Temperature coefficient—Voltmeter—Wheatstone bridge—Rheostats—Potentiometer—Earth resistance—Potential—Line faults

## CHAPTER II

CONDENSERS AND ELECTROSTATICS . . . . . 42

Condensers—Dielectrics—Condensers in series and parallel—Energy stored in condenser—Measurements of capacitance—Condensers in a.c. circuits—Electric charges—Electric flux—Capacitance of condenser—Electrostatic voltmeters—Insulation resistance of condenser

## CHAPTER III

ELECTROMAGNETICS—THE MOTOR EFFECT . . . . 79

Magnetic fields—The magnetic circuit (air core)—Forces on currents in magnetic fields: straight wires; coils; an experimental apparatus—Forces between currents: the ampere balance—Magnetic materials and magnetism—The magnetic circuit (general)—Permanent magnets—Moving coil instruments—Moving iron instruments—Magnetic pull—Pole strength and the magnetometer

## CHAPTER IV

PAGE
ELECTROMAGNETICS—THE GENERATOR EFFECT . . . 115

Electromotive force—Magnetic induction: Coils; straight conductor—Alternating e.m.f.'s—Inductance; mutual; self—Inductance in d.c. circuits: Growth and decay of currents; energy stored in magnetic fields; energy wasted in hysteresis—Inductance in a.c. circuits

## CHAPTER V

CHEMICAL ACTION AND ELECTRIC CURRENTS . . . 152

Primary cells—Secondary cells—The lead-acid cell—The alkaline or nickel-iron cell

## CHAPTER VI

THE ELECTRON THEORY . . . . . . . 177

Electrolysis—Constitution of the atom—Temperature effects—Contact p.d.—Metallic conduction—Magnetism—Emission of electrons, photo-electric, secondary, thermionic.

APPENDIX ON UNITS . . . . . . . . . 189

INDEX . . . . . . . . . . 197

# FOUNDATIONS OF TECHNICAL ELECTRICITY

## CHAPTER I

## ELECTRIC CURRENTS

1. **Conductors and Insulators.** Everyone knows something of the three uses of electricity, in electro-plating, in electric fires, and in electric motors. In these, in order, are used the three main effects of an electric current, that is, the chemical effect, the heating effect, and the magnetic effect. Other uses of these effects are—

| | |
|---|---|
| *Chemical Effect.* | Plating. |
| | Primary cells. |
| | Secondary cells (accumulators). |
| | Quantity meters (electrolytic). |
| *Heating Effect.* | Electric fires. |
| | Electric irons. |
| | Electric furnaces. |
| | Electric lamps. |
| *Magnetic Effect.* | Motors. |
| | Dynamos and alternators. |
| | Lifting magnets. |
| | Telephones, receivers, and loud speakers. |
| | Bells. |

In all of these cases the electricity is in motion, and electricity in motion is called an *electric current.*

Electricity at rest was known to the ancients. It was known that amber and jet when rubbed would attract light bodies, and that this attraction was due to electricity being produced in the amber or jet by the rubbing. But it was not until the beginning of the nineteenth century that electric currents were produced by chemical action, and not until the middle of the nineteenth century that electric currents were produced by magnetic action.

It is also a matter of common knowledge that in order that an electric current may flow there must be a *complete circuit.* A lamp does not light until a switch is closed.

The switch is a device by which a gap in the electric wiring may be bridged metallically or left open at will. There are two terminals from the electric supply, and the electric current may be regarded as coming from one terminal, flowing round the circuit provided (the circuit in this case including a lamp) and returning to the other terminal. The current is conveyed by wires, which are accordingly called *conductors* of electricity, and is prevented from leaving the wires by the *insulation,* as the covering on the wires is called. The material of which the insulation is made does not allow an electric current to flow through it; it is a non-conductor of electricity.

The distinction between conductors and non-conductors or *insulators* is not, however, absolute. All materials conduct electricity to some extent, but metals conduct enormously better than rubber or porcelain or ebonite, so that the former are commonly called conductors and the latter insulators. All conductors are not equally good. A list of conductors (and insulators) in order of merit is given below. The figures in the second column against each are proportional to their ability to conduct; the figures in the first column are the reciprocals of these figures and are proportional to their inability to conduct. This inability to conduct is known as the *resistivity* of the material.

It is seen that the ratio of the conducting figure for copper to that for ebonite is $5 \cdot 62 \times 10^5 \div 5 \times 10^{-16} = 1 \cdot 12 \times 10^{21}$; that is, copper is over a thousand million million million times better a conductor than ebonite; to all intents and purposes ebonite does not conduct at all.

2. **Electrolysis.** In all these cases the current flows through the conductor without any chemical action taking place. Some liquids, such as paraffin oil, are nearly completely non-conductors, i.e. almost perfect insulators, but solutions of certain salts and acids conduct electricity quite readily, and the flow of the electricity is accompanied by a chemical action. Suppose, for instance, that electricity is caused to flow through

a bath of copper sulphate by inserting into the bath two plates of platinum, separated from each other, and connected to a suitable electric supply. It will be found that a film of copper is deposited on one plate and bubbles of oxygen are liberated at the other. The copper sulphate has been split up into *ions* of Cu and $SO_4$, and these ions are caused to travel in opposite directions; the Cu ion is deposited on one platinum plate, which is called the *cathode*, and the $SO_4$ ion at the other platinum plate, called the *anode*, combines with water and liberates oxygen. This phenomenon is known as *electrolysis*, and the liquid as the *electrolyte*. The apparatus used for the experiment, i.e. the vessel containing the electrolyte and the plates, is known as a *voltameter*. If the connections to the

TABLE I

| | Resistivity Ohms per cm. cube | Conductivity Mhos per cm. cube |
|---|---|---|
| Silver . . . | $1{\cdot}66 \times 10^{-6}$ | $6{\cdot}03 \times 10^{5}$ |
| Copper . . . | $1{\cdot}78 \times 10^{-6}$ | $5{\cdot}62 \times 10^{5}$ |
| Gold . . . | $2{\cdot}12 \times 10^{-6}$ | $4{\cdot}14 \times 10^{5}$ |
| Aluminium . . | $3{\cdot}21 \times 10^{-6}$ | $3{\cdot}12 \times 10^{5}$ |
| Tungsten . . . | $5{\cdot}0 \times 10^{-6}$ | $2{\cdot}00 \times 10^{5}$ |
| Zinc . . . | $6{\cdot}1 \times 10^{-6}$ | $1{\cdot}64 \times 10^{5}$ |
| Brass . . | $6{\cdot}6 \times 10^{-6}$ | $1{\cdot}52 \times 10^{5}$ |
| Platinum . . . | $11{\cdot}0 \times 10^{-6}$ | $9{\cdot}10 \times 10^{4}$ |
| Tin . . . . | $11{\cdot}3 \times 10^{-6}$ | $8{\cdot}85 \times 10^{4}$ |
| Nickel . . . | $11{\cdot}8 \times 10^{-6}$ | $8{\cdot}48 \times 10^{4}$ |
| Iron . . . | $13{\cdot}9 \times 10^{-6}$ | $7{\cdot}20 \times 10^{4}$ |
| Steel . . . | $19{\cdot}9 \times 10^{-6}$ | $5{\cdot}03 \times 10^{4}$ |
| German silver . . | $16{\cdot}40 \times 10^{-6}$ | $6{\cdot}2-2{\cdot}5 \times 10^{4}$ |
| Platinoid . . . | $34{\cdot}4 \times 10^{-6}$ | $2{\cdot}91 \times 10^{4}$ |
| Manganin . . . | $44{\cdot}0 \times 10^{-6}$ | $2{\cdot}28 \times 10^{4}$ |
| Gas carbon . . | 0·005 | 200 |
| Silicon . . . | 0·06 | 16·7 |
| Gutta-percha . . | $2 \times 10^{9}$ | $5 \times 10^{-10}$ |
| Glass (soda-lime) . | $5 \times 10^{11}$ | $2 \times 10^{-12}$ |
| Ebonite . . . | $2 \times 10^{15}$ | $5 \times 10^{-16}$ |
| Porcelain . . . | $2 \times 10^{15}$ | $5 \times 10^{-16}$ |
| Sulphur . . . | $4 \times 10^{15}$ | $2{\cdot}5 \times 10^{-16}$ |
| Mica . . . | $9 \times 10^{15}$ | $1{\cdot}1 \times 10^{-16}$ |
| Paraffin wax . . | $3 \times 10^{18}$ | $3{\cdot}3 \times 10^{-19}$ |

terminals of the supply are reversed, the plates at which the copper and the oxygen appear are also reversed. If the current has been flowing for some time before the reversal of connections takes place, so that copper has been deposited on one plate, this copper will gradually disappear as copper is deposited on the other plate. The copper has combined with the $SO_4$ ions to form copper sulphate again. The chemical action has been reversed by the reversal of connections. If the reversal of connections takes place continuously and rapidly no chemical action will be apparent.

3. **Supply.** This brings us to a consideration of the supply. Some sources of electricity or "supplies" cause the electric current to flow in the same direction all the time. These are called *direct current* (d.c.) sources; examples are the dynamo, the wet or dry cell, and the accumulator. Other sources cause the electric current to flow first in one direction and then in the other, the change-over occurring many times a second. These are called *alternating current* (a.c.) sources; the chief example is the alternator.

Electrolysis will be apparent with a direct-current supply only. With an alternating current supply the effect is exactly the same as with a direct current supply of which the connections to the terminals are continuously reversed, i.e. the effect is nil.

The two terminals of a direct current supply are known as the *positive* (+) and the *negative* (—) respectively and are frequently marked accordingly. In the electrolysis of copper sulphate considered above, the oxygen appears at the plate connected to the positive terminal and the copper at the plate connected to the negative terminal. Electricity is looked upon as flowing from the positive terminal through the electrolyte to the negative terminal; and the copper moves with the electric current. This is always true in electrolysis; the metal moves with the current.

4. **Quantity and Current.** The weight of copper deposited on the cathode depends upon the *quantity of electricity* passed through the electrolyte; the two are in fact directly proportional to each other. If twice the quantity of electricity flows, twice the weight of copper will be deposited, and so on. The quantity of electricity that has passed may in consequence be measured by the weight of copper that has been deposited.

The practical unit of quantity of electricity is the *coulomb*, and this quantity causes 0·0003295 grams of copper to be deposited. If the electric current is steady or unchanged during any interval of time, its strength may be determined by dividing the total quantity that flows by the time; in the same way that the velocity of a body which has moved uniformly is determined by dividing the total distance covered by the time taken. The unit in which current strength would be measured is therefore the *coulomb per second*, but for convenience this unit is called an *ampere* instead of a coulomb per second. If a steady current flows through an electrolyte and $x$ grams of copper are deposited in $t$ seconds, the current strength is

$$\frac{x}{0{\cdot}0003295\, t} \text{ amperes}$$

In this way, by weighing the cathode before a current flows and after it has been flowing for a certain time, the current strength in a d.c. circuit may be measured.

Another unit of quantity of electricity in common use is the *ampere-hour*, i.e. the quantity which has flowed when a current of one ampere flows for one hour. Evidently, since an ampere is a coulomb per second, one ampere-hour is equal to 3 600 coulombs.

If the current strength in amperes is denoted by $I$, the quantity in coulombs by $Q$, the time in seconds by $t$, and the weight of copper deposited in grams by $W$, then the above relationships may be expressed algebraically by

$$Q = It \text{ or } I = Q/t,$$

and

$$W = zQ = zIt$$

where $z$ is a constant $= 0{\cdot}0003295$ in the case of the electrolysis of copper sulphate.

5. **Electrochemical Equivalent.** The constant $z$, that is, the weight of metal deposited by one coulomb, depends upon the electrolyte. If a solution of silver nitrate is the electrolyte, $z$ is 0·0011183 grams per coulomb, the metal deposited being, of course, silver; if it is zinc sulphate, 0·00033698 grams of zinc is deposited by one coulomb. The constant $z$ is known as the *electrochemical equivalent* of the ion; it is measured in grams per coulomb.

Although pure water is almost a non-conductor of electricity, a very small amount of acid or salt in solution will make it conduct quite readily, and the electrolysis of water is easily achieved. The most interesting form of voltameter is shown in Fig. 1.

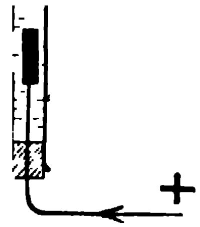

FIG. 1. ELECTROLYSIS OF WATER

The apparatus is filled with water slightly acidulated with, say, sulphuric acid ($H_2SO_4$). The platinum electrodes are near the bottom of the side tubes; the centre tube serves to fill the apparatus, the taps at the tops of the side tubes being opened to allow the air to escape. If the electrodes are connected as shown, the right-hand one to the positive terminal of a suitable d.c. supply and the left-hand one to the negative terminal, it will be found that gas is liberated at each electrode and bubbles up the side tubes to collect at the tops and force the electrolyte down the tubes and up the centre tube. It will further be observed that the volume of gas which collects in the left-hand tube from the cathode is exactly twice the volume of gas which collects in the right-hand tube from the anode, and if the gases are allowed to escape by opening the taps and the usual chemical tests are applied, it will be found that the gas in the left-hand tube is hydrogen, and that in the right-hand tube is oxygen.

The current moves the hydrogen ions 2H to the left and the sulphion $SO_4$ to the right; at the anode the sulphion $SO_4$ combines with water $H_2O$ to reform sulphuric acid $H_2SO_4$ and displace oxygen O. Thus the volume of hydrogen liberated is twice that of oxygen, and the amount of sulphuric acid remains unchanged; it is actually the water that has been decomposed by the electric current.

The weights of the hydrogen and oxygen liberated are, as in the case of metals, proportional to their electrochemical equivalents.

6. **Potential Difference.** When anything is set in motion a force must have been exerted. Nothing starts to move unless it is pushed. The push may be the force due to gravity, as in the case of water falling from a height or flowing in a river, or a vehicle rolling downhill, or the force due to the engine of a motor-car which drives the car uphill against gravity and friction, or the suction exerted by a fan which circulates air, and so on. In the same way electricity does not flow unless it is pushed.

The flow of electricity in a circuit is in many ways similar to the flow of water in a pipe. Imagine (Fig. 2) a large tank of water at a height above the earth and imagine a pipe connected to the bottom of the tank and reaching a second tank on the ground. Water will flow through the pipe from the first tank to the second tank. It is pushed through the pipe by the force due to gravity, and the strength of the push is proportional to the vertical distance between the tanks. Another way of stating the same thing is that the water flows because of the difference of water pressure between the ends of the pipe, and that this difference of pressure exists because of the difference of the potential energies of the water in the two tanks. Whenever this pressure difference or potential energy difference exists, a flow of water will take place if a suitable channel is provided.

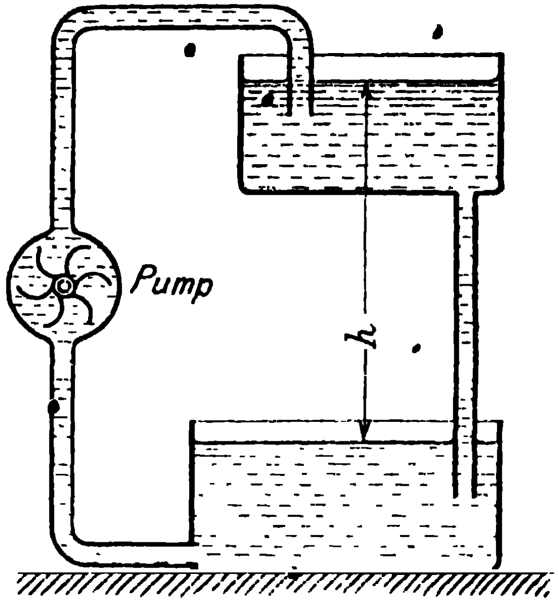

FIG. 2. WATER ANALOGY

In order that the flow may be continuous irrespective of the size of the tanks, a pump must be provided to raise the water from the lower to the upper tank.

In a similar manner electricity will flow through an electric circuit if a difference of electrical pressure, or as it is more commonly called, a *potential difference*, is maintained between the terminals of the circuit. The electrical potential difference corresponds with the water pressure, the electricity with the water and the electric current with the water current, the terminals of the electric supply with the water tanks, and the machinery at the generating station or the accumulator or primary cell with the pump. The electric counterparts of the tanks are, however, generally very small; so small that in effect there is no accumulation of electricity at the terminals as there is of water in the tanks; the water pipe is continuous instead of being terminated by tanks.

In the same way that the water flows from a point at a high pressure to one at a lower pressure, so electricity is to be looked upon as flowing from a point at a high potential to one at a lower potential. The potential difference between one point and another is positive if electricity flows from the first to the second.

The electrical potential difference (very often abbreviated to p.d.) is measured in *volts* (abbreviation, V.).

If the pipe is horizontal gravity plays no part, and the water is moved by the pressure difference at its ends. The work done in moving the water is the force multiplied by the distance moved through. If the pressure difference is $p$, the cross-sectional area of the pipe is $a$, and the length of the pipe is $l$, the force is $pa$, the distance moved through is $l$, and the work done is $pal$. But $al$ is the quantity of water moved, and $p$ is the pressure difference against which it is moved. Hence

$$\left\{\begin{array}{c}\text{Work}\\\text{done}\end{array}\right\} = \left\{\begin{array}{c}\text{Pressure}\\\text{difference}\end{array}\right\} \times \left\{\begin{array}{c}\text{Quantity of}\\\text{water moved}\end{array}\right\}$$

Similarly in the electrical case,

$$\left\{\begin{array}{c}\text{Work}\\\text{done}\end{array}\right\} = \left\{\begin{array}{c}\text{Potential}\\\text{difference}\end{array}\right\} \times \left\{\begin{array}{c}\text{Quantity of}\\\text{electricity moved}\end{array}\right\}$$

or, in an equation of units,

$$\text{joules} = \text{volts} \times \text{coulombs}$$

where joules, volts and coulombs are the units of work, p.d and quantity in order. (The common abbreviations for joule and coulomb are J. and C.) Thus the potential difference between two points of a circuit is the work done per coulomb moved from one point to the other. Also, the volt may be defined as the p.d. existing between two points when one joule is required to move one coulomb from one point to the other.

The joule is a unit of mechanical work and is equal to $10^7$ ergs or roughly to the work done in lifting a weight of one pound a height of 9 inches.

The rate of doing work or the work done per second is called the *power*, and since

$$\text{work} = \text{p.d.} \times \text{quantity},$$

$$\begin{aligned}\text{power} = \text{work per sec.} &= \text{p.d.} \times \text{quantity per sec.}\\ &= \text{p.d.} \times \text{current}\end{aligned}$$

The rate of working of one joule per second is called a *watt* (abbreviation, W.), and as has been seen before, a coulomb per second is called an ampere (abbreviation, A.), so that corresponding to the (units) work equation

$$\text{joules} = \text{volts} \times \text{coulombs}$$

is the (units) power equation

$$\text{watts} = \text{volts} \times \text{amperes}$$

$$746 \text{ watts} = 1 \text{ horse power} = 33\,000 \text{ ft.-lb. per min.}$$

7. **Resistance and Ohm's Law.** All motion is accompanied by friction. In a motor-car there is friction between the moving parts of the engine and of the chassis, which is minimized by oiling and greasing, and between the tyres and the road, which enables the car to be driven. When a ball is thrown in the air or rolled on the ground there is friction between the ball and the air or ground. All friction is accompanied by heat. If a piece of wood is pushed along a table it will be found that both the wood and the table become hot. Work has been done on the wood and the work appears as heat. When water flows in a pipe there is friction between the water and the sides of the pipe and the pipe is heated. If the pipe is horizontal and the flow is steady, all the work done in moving the water appears as heat.

There is a definite relationship between the work done and the heat produced. It is

$$(\text{Work in joules}) \div 4{\cdot}2 = (\text{Heat in calories})$$

The calorie (abbreviation, cal.) is the heat required to raise the temperature of one gram of water one centigrade degree.

Heat, in calories, can therefore be determined by the rise of temperature of a measured quantity of water, and the calories can be converted to joules by multiplying by 4·2.

The flow of electricity is no exception to the rule about friction and generation of heat. Whenever electricity flows through a wire or through an electrolyte heat is produced. This is, of course, the phenomenon upon which electric lamps and electric fires depend. In the case of a wire all the work expended in driving the current appears as heat. The amount of heat in a given time depends upon the strength of the current, the size of the wire, and the material of which it is made. With the thin tungsten wire of the electric lamp the rate of heating is such as to raise the temperature to a very high figure; with the comparatively thick copper wires leading to the lamp the same current produces a temperature rise which is quite unimportant.

The energy that is expended in heat in a given time, that is to say, the power that is expended in heat, is proportional to the square of the current flowing. An analogy is available in hydraulics, where in steady motion the force ($f$) due to friction is proportional to the velocity ($v$) of the fluid in motion in a pipe, so that if $r$ is some constant of the nature of a mechanical resistance to motion,

$$f = rv$$

Since the work done on the body in moving it against friction is equal to the force times the distance $s$ moved through,

$$\text{work} = fs = rvs$$

and the power is the rate of doing work, i.e.

$$\text{power} = \frac{\text{work}}{\text{time}} = rv\frac{s}{t} = rv^2$$

Thus the power used in producing frictional heat in this case is proportional to the square of the velocity; similarly the electrical power used in producing heat in a wire is found by actual measurement to be proportional to the square of the current and it is possible to write therefore

$$\text{power} = RI^2$$

where $I$ is the current and $R$ a constant which is called the *resistance* of the wire. If $I$ is measured in amperes and the power is measured in watts, then the unit in which the resistance $R$ is measured is called the *ohm* (abbreviation, Ω., Greek capital letter omega).

Further, since, as has been seen, the power used in driving a current along a wire is equal to the p.d. ($V$) at the ends of the wire multiplied by the current ($I$) through it, the power besides being equal to $RI^2$ is also equal to $VI$. Thus $VI$ must equal $RI^2$

or $V = RI$
or $I = V/R$
or $R = V/I$

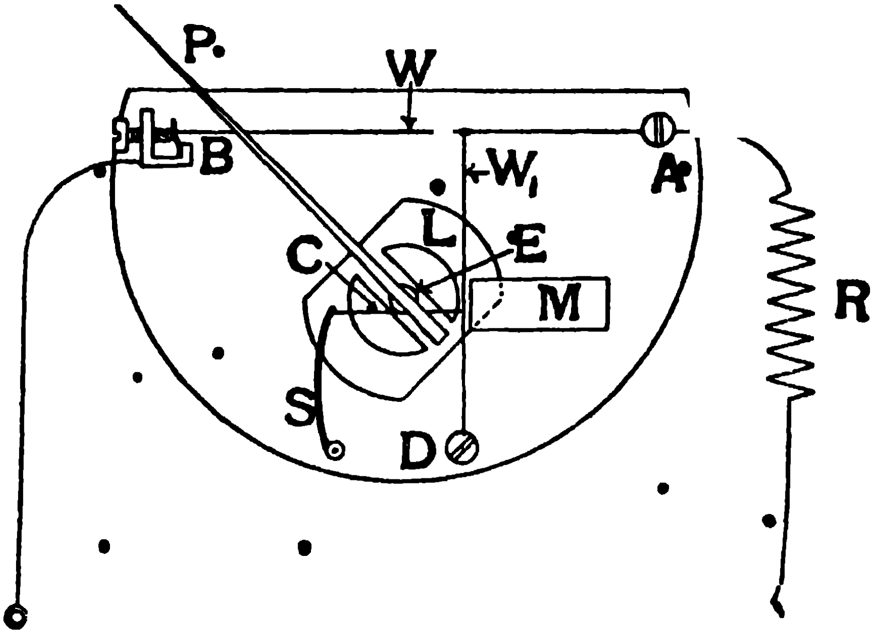

FIG. 3. HOT-WIRE AMMETER

This very important relationship is known as *Ohm's Law*. It may also be expressed in a units equation as

$$\text{amperes} = \frac{\text{volts}}{\text{ohms}}$$

8. **Hot-wire Ammeter.** When a wire is heated it lengthens and the increase in length can be used to measure the current that causes the heating. Steady temperature will be reached by the wire when the heat it loses per second by radiation, conduction and convection is equal to the heat it gains per second from the current; so the rise of temperature, and therefore the increase in length, will be proportional to the square of the current. A typical instrument is shown in Fig. 3. The current passes from $A$ to $B$ along the platinum-silver wire $W$ and heats it. To the wire is attached a second light wire $W_1$ fixed at $D$, and to the second wire a fibre $C$ attached to a spring $S$. The fibre $C$ is wound round a small pulley $E$ which

carries the pointer $P$. As the wire $W$ lengthens it is caused to sag by the spring $S$ acting through $Q$ and $W_1$, and the movement of the fibre rotates the pointer over a scale (not shown) marked in such a way that the pointer indicates the current in amperes. This instrument is known as a *hot-wire ammeter*.

9. **Alternating Currents and Potential Differences.** As with mechanical friction, the heat developed by an electric current does not in the least depend upon the direction of flow of the current. If the current is an alternating instead of a direct one, heat will still be developed and the instrument in Fig. 3 will still give a reading.

Let the instrument be calibrated with direct current by means of a voltameter, i.e. make a circuit consisting of the hot-wire ammeter ($A$), the copper voltameter ($V$) and a variable resistance wire ($R$) connected to a suitable source (d.c.) as shown Fig. 4. By altering the length of resistance wire (of which more later) the current $I$ through the circuit can be given different values, and for each value the current can be determined by the weight of copper deposited in the voltameter, and the indication of the pointer of $A$ marked with the current value.

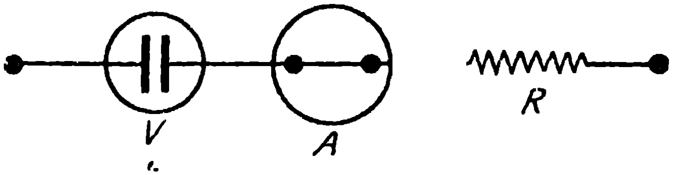

FIG. 4. CALIBRATION OF AMMETER

If now the d.c. source is replaced by an a.c. source, while no copper will be deposited in $V$, the ammeter pointer will still be deflected and the strength of the alternating current will be measured. If the alternations of the current were very very slow, the wire of the ammeter would have time to heat and cool as the current changed, and the deflections of the pointer would vary continually. But with all the alternating currents used in practical work, the variations are so rapid that the wire reaches an average temperature and the deflections of the pointer are steady. Since the rise of temperature is proportional to the square of the current, the extension of the wire is proportional to the mean square value of the current, and the calibration marks will be proportional to the root of the mean square value of the current. The *root mean square value*, always abbreviated to *r.m.s.* value, is what is meant when reference is made to the strength of an alternating current.

The current is an alternating current when the source is *a.c.*, i.e. when the source is such as to produce at the terminals an alternating potential difference. In a simple circuit consisting only of resistance, Ohm's law applies at any particular

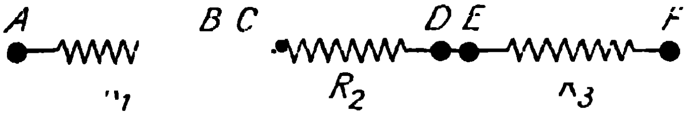

FIG. 5. RESISTANCES IN SERIES

instant of time. That is, if the current through the resistance $R$ at any instant is $i$ and the p.d. at that instant is $v$, then

$$v = Ri$$

It follows that if $I$ is the r.m.s. value of the current, the r.m.s. value of the p.d. $V$ is

$$V = RI$$

**10. Series and Parallel Connection.** All conductors have resistance, and sometimes conductors are put into a circuit

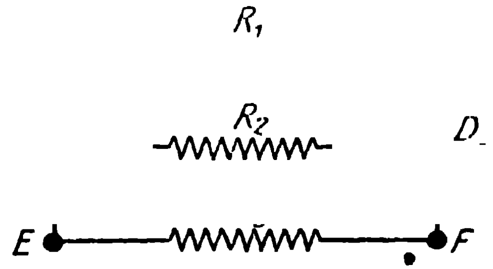

FIG. 6. RESISTANCES IN PARALLEL

simply because their resistance is required to reduce the current that flows from a given source. In this case the conductor is referred to as a *resistance*, or *resistor*, or *rheostat*.

There are two ways in which conductors may be connected; these are known as *series* connection (Fig. 5) and *parallel* connection (Fig. 6) respectively. In series connection, the end of one conductor is connected to the beginning of the next, and so on, and all the current flows through each conductor in turn; in parallel connection all the beginnings of all the conductors are connected together and all the ends are connected together, and the current divides and a part only flows through each conductor.

Certain definite relationships hold in these cases. In the series connection (Fig. 5) let the resistances of the conductors $AB$, $CD$, $EF$, . . . be $R_1$, $R_2$, $R_3$, . . . and let the potential difference between $A$ and $B$ be $V_1$, between $C$ and $D$ be $V_2$, between $E$ and $F$ be $V_3$, and so on. If the current through each of the conductors is $I$, by Ohm's law for the conductor $AB$, $I = V_1/R_1$, for $CD$, $I = V_2/R_2$, for $EF$, $I = V_3/R_3$, and so on, or

$$V_1 = IR_1, \quad V_2 = IR_2, \quad V_3 = IR_3$$
$$V_1 + V_2 + V_3 + \dots = I(R_1 + R_2 + R_3 + \dots)$$

i.e. the sum of the potential differences across the conductors in series is equal to the current multiplied by the sum of the resistances of the conductors. If the total p.d. across the conductors (from $A$ to $F$) is $V$ and the combined resistance of the conductors is $R$,

$$V = IR$$

and $$R = R_1 + R_2 + R_3 + \dots,$$

or the resistance of a number of conductors in series is equal to the sum of the resistances of the individual conductors.

In the case of parallel connection (Fig. 6), the p.d. ($V$) across each conductor is the same but the currents are different. Let the currents be $I_1$ in $AB$, $I_2$ in $CD$, $I_3$ in $EF$, etc.; then by Ohm's law

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \quad I_3 = \frac{V}{R_3} \dots$$

and, by adding, the total current $I$ is

$$\begin{aligned} I &= I_1 + I_2 + I_3 \\ &= V\left[\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots\right] \end{aligned}$$

If the current $I$ is written

$$I = \frac{V}{R}$$

where $R$ is the resistance of the combination of conductors in parallel, it is seen that $R$ is determined from the relationship

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$$

The current in each branch of a parallel circuit is frequently required. It is readily obtained as follows—

$$I_1 = \frac{V}{R_1} = \frac{IR}{R_1}$$

$$\text{i.e.} \quad \left\{ \text{any branch} \right\} = \left\{ \text{current} \right\} \times \left\{ \frac{\text{the combined resistance}}{\text{the resistance of the branch}} \right\}$$

A case that occurs very frequently is the one in which there are only two conductors in parallel. Then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

and

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

11. **Resistivity and Conductivity.** The resistance of a wire depends upon its length and its cross-sectional area. Since any wire can be looked upon as a number of shorter wires joined in series, it is evident that the resistance of a wire is directly proportional to its length. And since a wire can be looked upon as a number of thin wires of equal length joined in parallel, it is evident that the resistance of a wire is inversely proportional to its cross-sectional area. For if the cross-sectional area is $a$, and the wire is looked upon as divided into $a$ wires of unit cross-sectional area, each of resistance $r$, the combined resistance $R$ is given by

$$\frac{1}{R} = \frac{1}{r} + \frac{1}{r} + \ldots a \text{ times}$$

$$R = \frac{r}{a}$$

The resistance of a wire can accordingly be written

$$R = \frac{\rho l}{a}$$

If $l$ is equal to 1 cm. and $a$ is equal to 1 cm.$^2$

$$R = \rho$$

and it is evident that the constant $\rho$ (Greek letter rho) is the resistance of a wire 1 cm. long and 1 cm.$^2$ cross-sectional area. $\rho$ is different for different materials and is called the *resistivity* (or *specific* resistance) of the material. Its reciprocal is called the *conductivity* of the material, and is usually denoted by $\sigma$ (Greek letter sigma). Table I on page 3 gives values of the resistivity and conductivity of various materials, the first column in ohms per cm. cube and the second in reciprocal ohms, i.e. *mhos* per cm. cube.

12. **Temperature Coefficient.** The resistance of a metal conductor is found to be a true constant if the temperature of the metal is kept constant. Some other conductors which are not metal do not behave in this simple way; there is a variation of the resistance with current strength, and for these Ohm's law does not apply. However, to consider the important case of metals only, the resistance of any piece of metallic wire is found to be so constant at any given temperature that wire resistances properly constructed are most reliable electric standards. All pure metals increase in resistance with rise of temperature. For moderate rises of temperature, the increase of resistance is proportional to the rise, so that if the resistance is plotted to a temperature base as in Fig. 7, the graph is found to be a straight line. The increase in resistance is found to be proportional to the rise in temperature and to the original resistance. Taking the resistance $R_0$ at 0° C. as standard, the resistance $R$ at any other temperature $\tau$ (Greek tau) is

$$R = R_0 + R_0\alpha\tau = R_0(1 + \alpha\tau)$$

$\alpha$ (alpha) is the change of resistance per ohm for 1° C. rise in temperature and is called the *temperature coefficient of resistance*.

By solving this equation, it is found that

$$\alpha = \frac{R - R_0}{R_0\tau}$$

This shows how $\alpha$ may be calculated from observations of the resistance of a wire at various temperatures. The value of $\alpha$ for copper is 0·00428 per 1° C. so that a copper wire of which the resistance is 1 ohm at 0° C. would have a resistance at 10° C. of 1·0428 ohm and at 100° C. of 1·428 ohm. Table II shows the temperature coefficient of various metals and alloys.

This variation of resistance with temperature is relatively large, and as the value of the resistance of any conductor can be measured very precisely, it is possible to use the measurement of the resistance of a wire to find the temperature of the wire. This is the principle of the resistance thermometer and also of a very convenient method of measuring the temperature rises in the windings of electrical machinery due to the currents passing through them or to any other cause of heating. This heating must not be excessive; in usual cases the increase of resistance allowed is about 20 per cent.

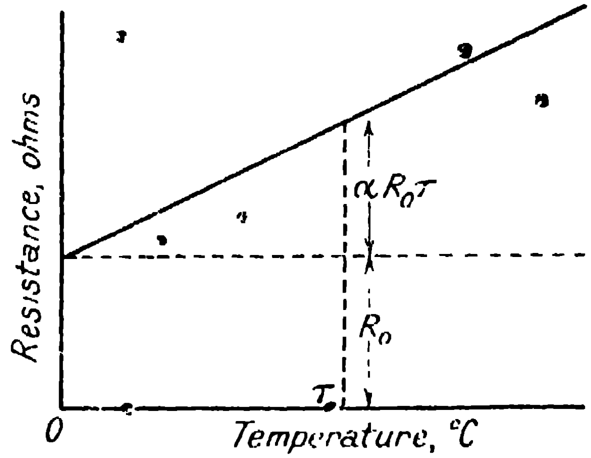

FIG. 7. INCREASE OF RESISTANCE WITH SMALL TEMPERATURE CHANGES

When the variation of resistance of a wire is used for measuring high temperatures, it is found that the graph of resistance to a temperature base is not quite straight; it is slightly curved as in Fig. 8. This deviation from a straight line is found to be proportional to the square of the temperature rise. This is shown in the figure and is expressed in the formula by the term $\beta T^2$

$$R = R_0 (1 + \alpha T + \beta T^2).$$

## TABLE II

### TEMPERATURE COEFFICIENTS

α = per °C. per ohm at 0° C.

| Metal | α | Alloy | α |
|---|---|---|---|
| Aluminium | 0·0038 | Brass | 0·0010 |
| Copper | 0·00428 | Constantan (Eureka) | − 0·00004 to + 0·00001 |
| Gold | 0·0040 | | |
| Iron, pure | 0·0062 | | |
| Steel | 0·0016 to 0·0042 | German silver | 0·00023 to 0·0006 |
| Lead | 0·0043 | Manganin | 0·000002 to 0·00005 |
| Platinum | 0·0038 | | |
| Silver | 0·0040 | | |
| Tin | 0·0045 | Platinum-Silver | 0·00024 to 0·00033 |
| Tungsten | 0·0051 | | |
| Zinc | 0·0037 | | |

$\beta$ (beta) is generally only a small number, but there is an exception in the case of iron. The resistance of an iron wire increases rapidly with temperature when it is red hot. If a fine iron wire is enclosed in a bulb filled with a gas which does not act on hot iron and a current of gradually increasing strength is passed through the wire, the temperature of the

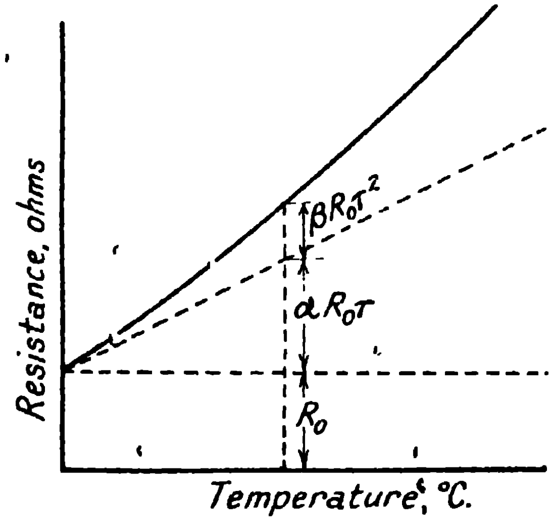

FIG. 8. INCREASE OF RESISTANCES WITH LARGE TEMPERATURE CHANGES

wire increases as the current increases and in consequence its resistance also. Therefore the p.d. required to push the current through the wire increases very rapidly with the current and at a certain value of the current the p.d. required may vary over quite a wide range with only a small change of current. This behaviour is shown in Fig. 9, which is a graph of measurements made with such a wire adjusted to get to the right temperature for 0·25 ampere. This arrangement is called a *barretter*, and is used where it is required to maintain a constant current in a circuit in spite of fluctuations in the voltage of the supply or in the resistance of the rest of the circuit.

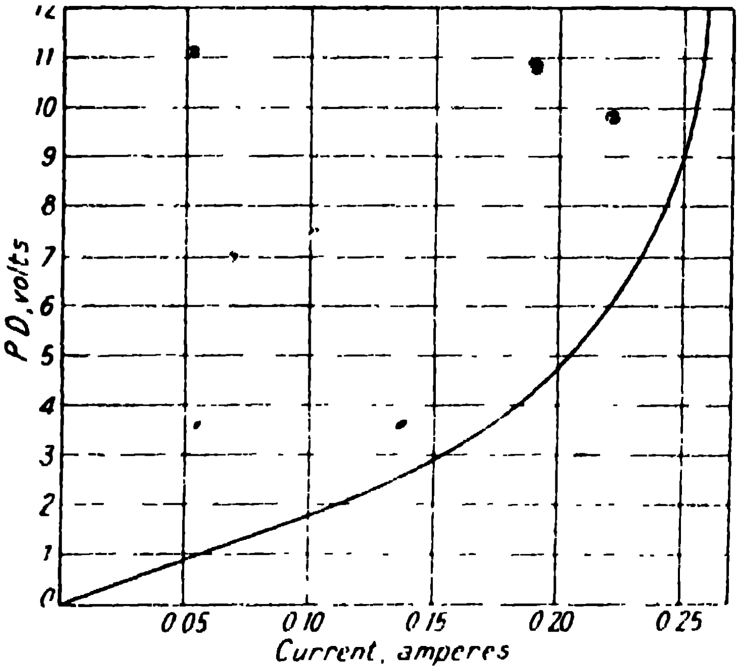

FIG. 9. P.D.—CURRENT CURVE OF BARRETTER

While the temperature coefficient of most pure metals is practically the same, that for alloys may be much less, and for carbon it is actually negative; the resistance decreases with temperature. For such alloys as manganin and Eureka, which have a very high resistivity, the temperature coefficient is very small, so that the resistance changes are quite negligible with the temperature changes normally taking place in a laboratory. Wires made of these alloys are accordingly used in making resistors to be used as standards for the measurement of unknown resistances.

**13. Voltmeter.** If the resistance of a circuit is known and the current through it is measured, the potential difference across the circuit can be calculated at once from Ohm's law, $V = RI$. For instance, if the resistance of a lamp is 800 ohms and a current of a quarter of an ampere flows through it, the potential difference across the lamp is 200 volts.

Potential difference may be measured in this way by connecting a known resistance in series with an ammeter (also of known resistance) across the two points whose potential difference is to be measured. In the diagram (Fig. 10) the straight

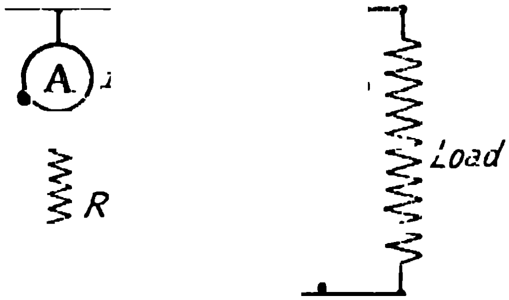

FIG. 10. PRINCIPLE OF VOLTMETER

lines are supposed to be conductors without resistance, the heavy black circles represent the terminals of the supply, the circle with an *A* inside represents the ammeter, and the zig-zag lines the resistances. The load resistance may be lamps or motors or fires, any device in fact which is being run off the supply.

If the ammeter resistance is $r$ ohms, the series resistance is $R$ ohms, the current flowing through the ammeter is $I$ amperes, and the p.d. of the supply is $V$ volts, then

$$V = I(R + r)$$

and $V$ is at once found if $R$ and $r$ are known, as $I$ is measured by the ammeter.

Usually the resistance $R$ is located in the case containing the instrument and included in the circuit within the instrument between its terminals, and the scale is marked in volts instead of in amperes. The instrument is now called a *voltmeter*. Different voltage ranges are readily obtained by including different resistors $R$ connected to separate terminals.

An ammeter measures the current which passes through it, and its indication does not depend on the resistance of the circuit. Consequently, the accuracy of an ammeter is independent of changes of temperature. When it is used in the way just described in order to measure a p.d., the resistance in circuit must not vary with temperature noticeably, otherwise the same p.d. would not send the same current through the instrument, and therefore would not give the same reading when the temperature changes. The resistance $R$ in series must be made of an alloy whose resistance does not change with temperature, and the resistance, $r$, of the ammeter must be small enough in comparison for any change in the value of $r$ to be negligible.

14. **Wheatstone Bridge.** A wire for which $R$ is known to be accurately one ohm may be used as a standard to measure the resistances of other wires, if some way of comparing them can be devised. The National Physical Laboratory in England and similar standardizing laboratories in other countries have found that an ohm is the resistance of the mercury in a tube 1 mm.$^2$ in area and 106·3 cm. long at 15° C., so that a resistor of resistance exactly one ohm can readily be made with glass tubing and mercury. This is the legal standard of resistance. To compare another resistance with a standard, the property which describes and defines the ohm is used, i.e. the p.d. between the ends of a resistor when carrying a current. With a uniform wire carrying a current, the p.d. is proportional to the length. The p.d. on each centimetre is the same fraction of a volt, so if the wire is stretched along a scale and the current is kept steady the potential difference between one end of the wire and any point along the wire can be read on the scale.

If the same supply is fed to a length of uniform wire $AB$ and to two resistances in series—standard $R$ and unknown $X$, Fig. 11—there is, of course, the same p.d. between the ends $AB$ and $CD$ and the same potential fall along $CD$ as along $AB$. Some point $Q$ on $AB$ will be at same potential as $F$, the junction of $R$ and $X$. If $Q$ can be found, the ratio of the potential drops on $R$ and $X$ will also be known, for it will be the ratio between $l_1$ and $l_2$, the lengths $AQ$ and $QB$ respectively. The ratio of the p.d.'s on $R$ and $X$ is the ratio of their resistances, since the same current flows through each; thus

$$\frac{X}{R} = \frac{l_2}{l_1}$$

or

$$X = \frac{l_2}{l_1}R$$

By this method any resistance can be compared with the standard. The point $Q$ is found by taking a sensitive galvanometer* and joining one terminal to $F$ and the other to a wire which is touched on $AB$ at various points. If $F$ and $Q$ are at the same potential no current will flow through the instrument. If $Q$ is too near $A$, its potential will be higher than the potential of $F$ and electricity will flow from $Q$ to $F$; if $Q$ is too near $B$, current will flow from $F$ to $Q$ through the galvanometer. A few trials enable the balance to be found exactly. The galvanometer bridges across from $AB$ to $CD$, and so this method is called a *bridge* method. It was used a great deal by Professor Wheatstone, and is generally called the *Wheatstone Bridge.*

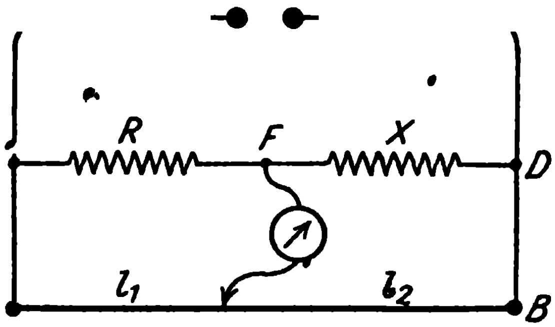

FIG. 11. WHEATSTONE BRIDGE

If the value of $X$ is very different from that of $R$, say $R = 100X$, then $Q$ is near one end of the wire $AB$, and it is not possible to make an accurate measurement. A very small error in reading the position of $Q$ on the scale would make a large error in the calculated value of $X$. To use the slide wire bridge properly, it is necessary to have a set of standards of different values so that one whose value is near that of the resistor to be measured can be chosen. Such sets of standard resistors, in the form of coils of wire, are made up in boxes arranged so that any desired resistance coil can be chosen.

A very usual arrangement of such a box is shown in Fig. 12. The top consists of a stout ebonite plate. On this is fixed a set of brass bars separated by narrow gaps. The wire coils

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*A *galvanometer* is a sensitive detector of direct currents which indicates the direction as well as the strength of the current. It is described in Chapter VII.

forming the resistances are connected, one across each gap. The gap can be bridged by a tapered plug which fits accurately into a tapered socket which is formed by the two brass blocks, one half of the socket being formed in each block. When the plug is in the socket the corresponding coil is short-circuited by the plug, and the effective resistance is zero. When the plug is out, the resistance of the coil is in circuit. The coils are arranged in series, and different sets of resistance coils

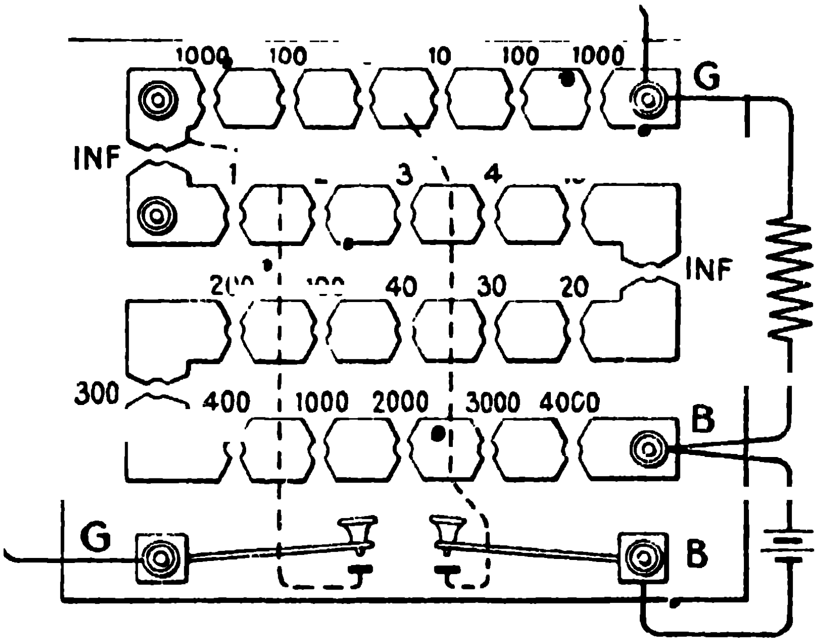


FIG. 12. POST OFFICE BOX

may be chosen by removing the corresponding plugs. The other plugs should be pushed in to make good contact. The coils may be 1, 2, 3, 4, 10, 20, 30, 40, 100, 200, 300, 400, 1 000, 2 000, 3 000, 4 000, or 1, 2, 2, 5, 10, 20, 20, 50, 100, 200, etc., ohms. From either set coils can be chosen to make up any value of resistance up to 11 100 ohms in steps of 1 ohm. The coils themselves are made up from insulated wire and wound on insulated bobbins. The winding is most often done by doubling the wire and wrapping the doubled wire on the bobbin, so that the current when passing through the coil flows round the bobbin clockwise for half the length of wire and counterclockwise for the other half of the length. The reason for this will be clear when the inductance of a coil has been studied. With such a resistance box the balance point can easily be brought near the centre of the bridge by taking out the proper plugs. In fact, it is easy to find two resistances in the box, differing by one ohm only, such that the balance for one is on one side of the centre and the other on the other side. The unknown resistance then lies between these two resistances.

The same information could be obtained if the two halves of the slide wire were wound into coils like those in the box and the mid-point were joined to a terminal. This would be a much more compact arrangement than the slide wire bridge, and there would be no loose pieces. With such an arrangement the value of any resistance to the nearest ohm up to 11 110 ohms would be found. The range of measurement is much extended if, instead of replacing the slide wire by a pair of equal coils, it is replaced by two sets of three coils, 1 000, 100 and 10 ohms, in each set. By taking different pairs of these, one from each set, ratios of 100 : 1, 10 : 1, 1 : 1, 1 : 10, and 1 : 100 can be obtained, so that the equation

$$\frac{X}{R} = 10^n \text{ and } X = R \times 10^n$$

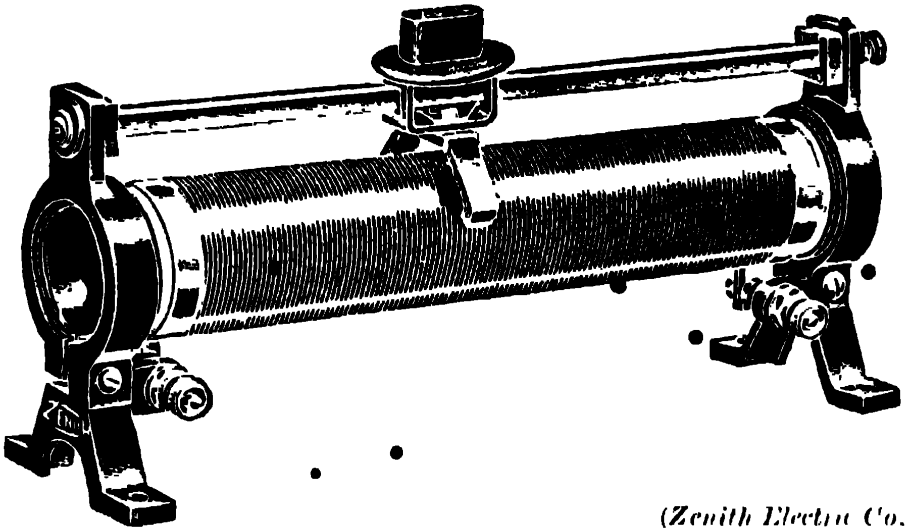
where $n = 2, 1, 0, -1$ or $-2$ replaces the equation

$$\frac{X}{R} = \frac{l_2}{l_1}$$

In this way the range of measurement is extended up to 100 times greater and down to 100 times smaller than with a ratio of equality. Also the value of the resistance measured is obtained by merely moving the decimal point in the figures giving the resistance $R$ in the box.

A box containing the resistance coils in this arrangement is called a *Post Office Box*, as it was designed by Post Office engineers for the use of linesmen in testing telegraph lines. For very many years it has been the standard arrangement for general resistance measurements. There are generally included on the ebonite panel two switches for the battery and galvanometer respectively, and terminals for connections to the battery, galvanometer and the resistance to be measured. The arrangement of the panel varies considerably with different makers, but the proper connections are either clearly marked or easily traced. Fig. 12 shows the top of a typical box and the scheme of connections lettered to correspond.

15. **Rheostats.** It is often necessary to adjust the current in a circuit to a particular value, and this is generally most



(*Zenith Electric Co.*)

FIG. 13. SLIDING CONTACT RHEOSTAT

conveniently done by adjusting the resistance of the circuit. For this purpose the arrangement shown in Fig. 13 is often used. Wire of a high resistance alloy is wound on a former of slate or an enamelled iron tube, and the ends of the wire are connected to two terminals. The turns are insulated from each other. A third terminal is connected to a metal bar which is fixed parallel to the axis of the former and carries a slider which makes contact with the windings. As much or as little of the winding as is desired can be included in the circuit, if the connections are made as in Fig. 14.

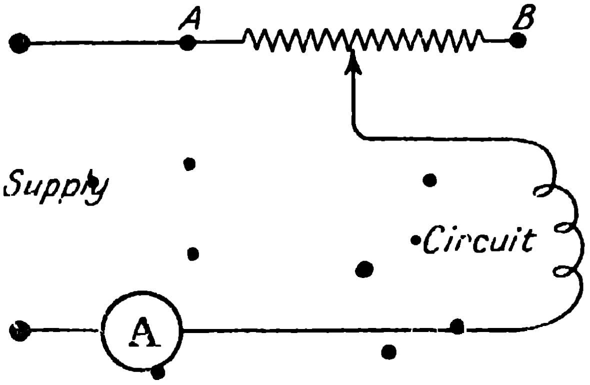


FIG. 14. USE OF RHEOSTAT TO VARY CURRENT

The value of the resistance in the circuit does not require to be known, as the position of the slider is adjusted till the required current as indicated by the ammeter *A* is obtained. The gauge of wire and the size of the tube are chosen so that the *rheostat*, as it is called, does not get too hot. The watts to be dissipated from the surface of the winding should not exceed about 0·4 watt per cm.² Variable resistances of this kind are sometimes called *potentiometers*, but this is an entirely wrong use of the name.

A rheostat with a terminal at each end and a slider making contact between them can be used as a potential divider when connected as in Fig. 15. The p.d. between $A$ and $C$ can be adjusted to have any value between zero and the value, $V$, maintained by the source between the terminals $A$ and $B$. No measurement of the potential difference is made by this arrangement.

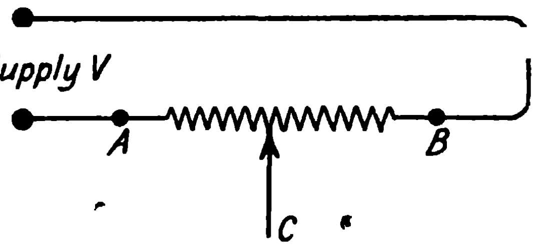


FIG. 15. USE OF RHEOSTAT AS POTENTIAL DIVIDER

16. **Potentiometer.** The potentiometer is a very accurate means of measuring p.d.'s. If a uniform wire is carrying a steady current, the fall of potential is the same in each centimetre of its length. This gives a scale of potential differences which can be used to measure and compare potential differences in other conductors. The general arrangement is shown in the circuit of Fig. 16.

$AB$ is a uniform wire stretched along a scale. It is carrying a constant direct current $I$ amperes supplied by the source. The strength of the current can be adjusted by means of the resistor $R$. If the resistance of each centimetre of the wire $AB$ is $r$ ohms, the p.d. on a length $AE = l$ cm. is

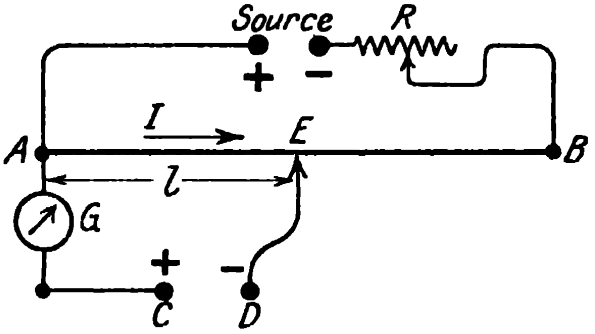


FIG. 16. POTENTIOMETER

$$Ilr \text{ volts}$$

Two wires are connected, one to $A$, and one to a movable contact $E$ on the wire. These wires will be at the same potential as the points on the wire $AB$ to which they are connected if no current is flowing in them. A sensitive galvanometer $G$ is connected in the lead from $A$ to detect any current. Now if $C$ and $D$, the other ends of these leads, are connected to two points whose p.d. is to be measured, no current will flow through the galvanometer if the p.d. on the wire is equal to the p.d. to be measured. If the p.d. $Ilr$ between $A$ and $E$ is greater than the p.d. between $C$ and $D$, current will flow from $A$ to $C$ and back by $D$ to $E$. If $Ilr$ is smaller, current will flow in the opposite direction. By adjustment of the length $l$ balance is obtained when no current flows, say at $l_1$ cm. for a p.d. between $C$ and $D$ of $V_1$ volts. The same operation is

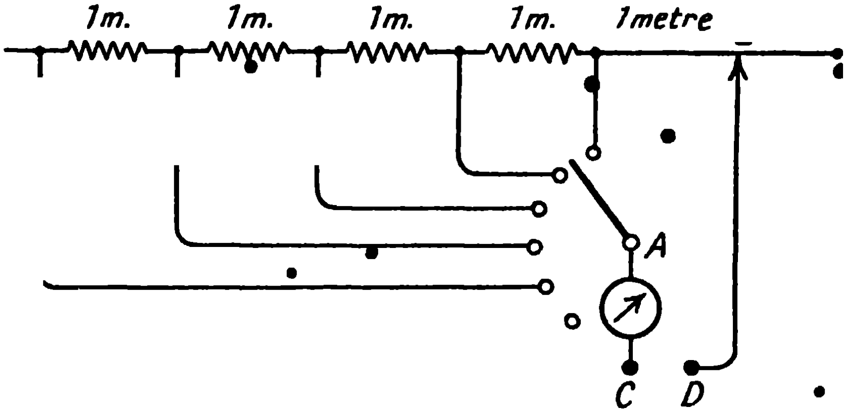


FIG. 17. POTENTIOMETER WITH EXTENDED RANGE

repeated for another p.d. $V_2$, and balance is obtained with a length $l_2$ cm.

Then
$$\frac{V_1}{V_2} = \frac{Il_1r}{Il_2r} = \frac{l_1}{l_2}$$

If either $V_1$ or $V_2$ is known, the other can be found.

It is convenient to make the scale of the wire $AB$ read directly in volts. This can be done by adjusting the strength of the current $I$. Standard cells of very constant p.d. are available. A cadmium cell, for example, which has a p.d. of 1·0183 volts, is connected at $CD$ and the slider set at 1·0183 m. = 101·83 cm., and the current $I$ is adjusted by the rheostat $R$ until the galvanometer $G$ shows no deflection. Each millimetre of the scale will then read 1 millivolt. As it would be inconvenient to have a very long scale the wire is partly made up into coils, each containing say one metre of wire exactly, and one metre only is stretched out along the scale. The ends of each coil are brought to a set of switch contacts as in Fig. 17. The contact arm of the switch then acts as the point $A$ in Fig. 16. The potentiometer is an exceedingly useful device, as it is equivalent to a voltmeter having a scale as long as the wire and divided uniformly.

In addition to measuring potential differences the potentiometer can be used to measure currents and resistances. The current to be measured is passed through a standard resistance which is known, say 1 ohm, $\frac{1}{10}$ ohm, etc., and the potential difference at its ends is measured by connecting them to the points $CD$. Resistances may be compared by passing a current through them in series and finding the ratio of the p.d.'s this current produces in them. This method is particularly useful for measuring low resistances which are to be used as shunts for heavy currents for ammeters. These have four terminals, two terminals are for attachment of the current leads and two, called *potential leads*, are for attachment to the instrument, see Fig. 18. The resistance value of the shunt is the resistance between the points where these are connected. The reason why they are made in this way is that it is impossible to make an electrical connection or contact of really zero resistance, and if a contact is carrying a large current the p.d. across the contact, which is $IR$, may be measurable even for a small resistance of contact $R$. In this case the position of insertion in the clamps $A$ and $B$ of the wires leading to the instrument would affect the reading. By arranging separate terminals $C$ and $D$ for the instrument leads, the resistances of the contacts $A$ and $B$ carrying the large current do not affect the reading. As the instrument current is small, the product $IR$ for the terminals $C$ and $D$ is negligible if they are connected with ordinary care.

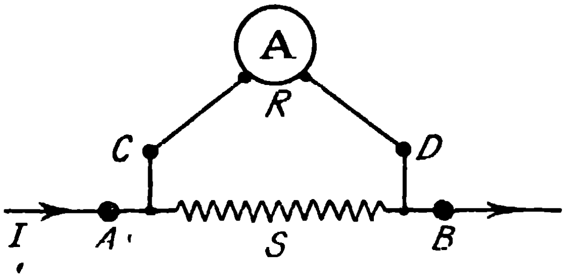


FIG. 18. AMMETER SHUNT

If the resistance of the shunt is $S$ ohms and of the ammeter is $R$ ohms, the current through the instrument is

$$\frac{S}{S+R} \times \text{(total current)}$$

The shunt $S$ can be made to have values which make the shunt factor $\frac{S}{S+R}$ any convenient factor such as $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{500}$, etc. The shunt thus converts the instrument into one with a higher range.

17. **Earth Resistance.** A consideration of the resistance of the Earth is of considerable importance. Although the resistivity of the Earth's surface varies very considerably according to the nature of the soil (the sea is a good conductor and dry sand very poor), the cross-sectional area taking part in any conduction of electricity between two conductors led into the earth at different points is so considerable that the actual resistance encountered is very small indeed, and almost entirely situated in the neighbourhood of the conductors.

This fact was discovered in the very early days of telegraphy. It was found that if, instead of sending current from one place to another over one wire and back over another, as in Fig. 19 (*a*), one terminal of the source and one terminal of the receiving apparatus *R* were connected to plates buried in the earth, and only one wire was run from the source to the apparatus, as in (*b*), nearly twice the current was received in the apparatus. Instead of the resistance of the two wires in series in (*a*), there is in (*b*) the resistance of one wire plus the resistance of the Earth, and the latter was found to be negligible. This arrangement is known as an *earth return* circuit.

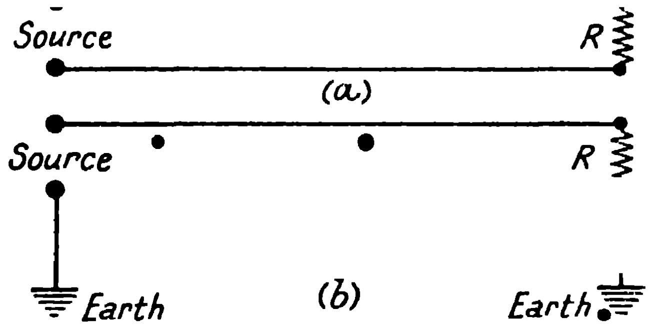


FIG. 19. EARTH RETURN CIRCUIT

The current from one buried plate to the other must be regarded as flowing in stream lines somewhat as indicated in Fig. 20. If two conductors, such as a pair of copper balls, are put into the ground at a distance from one another, and they are connected to the terminals of an electric supply, the current flowing from one to the other through the earth does not confine itself to the direct line joining them. It flows out of the + ball on all sides and into the − ball similarly from all sides; the current flows in curved lines from one to the other. These balls may be called *electrodes*.

The arrangement is really equivalent to many conductors connected in parallel. Each possible channel of flow will take its share of the current. If a small area is imagined to be drawn on one ball, the current from this patch will flow along a set of neighbouring paths in the ground, so that the lines of flow will form a bundle which ends up on a patch on *B*. This bundle keeps itself to itself all the way; no current strays out of it or into it, to or from other bundles of lines of flow which run alongside it. The current which starts from the path on *A* all arrives at its corresponding patch on *B* and can be enclosed in an imaginary tube in the ground. This tube is wider in some parts than others and the sides of it are lines along which the current flows. As the current is the same all along the tube, the current density, which is the current per cm.$^2$, is least where the tube is widest.

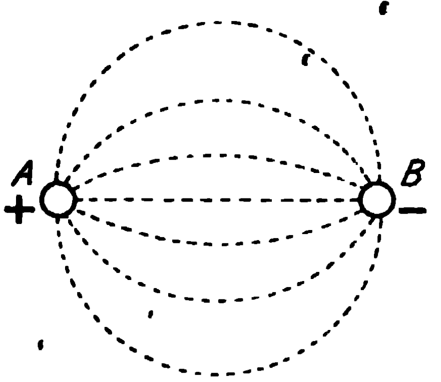


FIG. 20
CURRENT FLOW BETWEEN BURIED ELECTRODES

At any cross-section of the tube,

Current density × area = current in the tube

This current may be called the flow in the tube. The patch on the electrode may be any size at all. Let it be chosen so that one ampere flows from it. The flow in the tube is then unity and the tube may be called a unit tube. This may be done for the whole surface of the electrode *A*, and then the number of unit tubes leaving *A* is the total current from *A* to *B*, and must be the same as the number of unit tubes reaching *B*. If any area is imagined to be marked out underground, such as by a buried loop of string, the number of unit tubes passing through the loop is the current through the loop and may be called the flow through it.

If an imaginary surface is drawn in the ground enclosing the electrode *A*, the total flow through it is the same whatever the shape or size of the surface, for all the flow from *A* passes out through it. If the surface is a sphere of radius $r$ with its centre at $A$, the area of the surface is $4\pi r^2$ and the average flow per cm.$^2$ = $I/4\pi r^2$ if $I$ is the total flow.

Now if $B$ is very far away, the flow lines at $A$ are not affected by it and the flow is equal in every direction and is the same over the whole of the sphere. The flow per cm.$^2$ is then $I/4\pi r^2$, or the flow density is inversely proportional to the square of the distance from $A$. The number of unit tubes per cm.$^2$ is the flow per cm.$^2$ and so is $I/4\pi r^2$. The same result is obtained

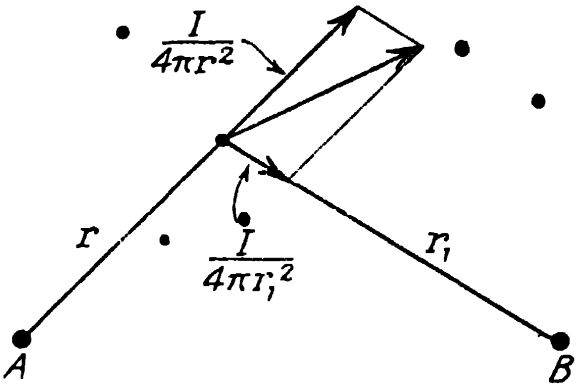


FIG. 21. FLOW AT ANY POINT BETWEEN ELECTRODES

if the flow inwards at $B$ is considered; the flow inwards per cm.$^2$ is $I/4\pi r_1^2$ if $r_1$ is the distance from $B$.

Where the flow of $A$ and $B$ affect one another we have to find the resultant flow by combining $I/4\pi r^2$ outwards from $A$, with $I/4\pi r_1^2$ inwards to $B$. The resultant flow is in a direction between the two, as in Fig. 21. It is in fact given in magnitude and direction by the same construction as that used in the parallelogram of forces. As the distance from an electrode increases the cross-section of the tube increases very rapidly, and as the resistance per cm. of a conductor is inversely proportional to its area, practically the whole of the resistance between the electrodes is close to the electrodes, where the area of the tube is small. If the soil in which the electrode is buried is a good conductor, the total resistance is very small.

18. **Potential.** Because of the negligible resistance between the two plates the potential difference between the plates is practically zero. In fact the Earth is of such vast extent and in general such a good conductor that never between points on its surface is there any serious potential difference. The earth is consequently very suitable as a reference point in

estimating p.d.'s, and when the p.d. between any point and the Earth is under consideration, it is very usual to speak simply of the *potential* of the point, meaning the p.d. between the point and earth. If current would flow from the point to earth, were a wire connected between the two, the potential of the point is said to be positive or higher than earth potential; if current would flow from earth to the point, its potential is said to be negative or lower than earth potential.

Very frequently one point (usually at the source) of a circuit,

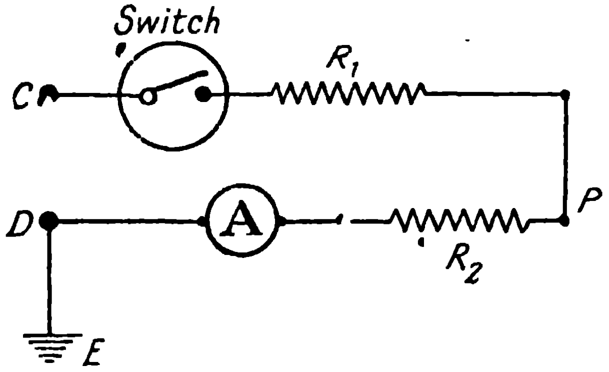


FIG. 22. CIRCUIT EARTH

*D* in Fig. 22, is connected to earth in order to prevent the p.d. between any part of the circuit and earth becoming (by any means) more than the p.d. at the terminals *CD* of the source. This is not an earth return arrangement, which is very little used to-day and then only for the small currents of telegraphy, but merely a safety device. Instruments, etc., should always be located next to the terminal connected to earth. Then if they are accidentally touched, no current will pass through the body to earth. The switch is placed between the apparatus and the high potential terminal so that when it is opened all the apparatus is at earth potential and there is no possibility of accident.

19. **Line Faults.** If the insulation fails at any part of a supply line, say at *P*, Fig. 22, and a connection is thus made to the earth, the part of the line *PD* between that point and the earthed terminal of the source is said to be *short-circuited* by the earth, as the current, instead of flowing through that part ($R_2$ in Fig. 22) of the circuit, flows through the earth which is now in parallel. If $R_2$ is the load—motors, lamps, etc. —which it is intended to supply and $R_1$, merely the resistance

of the electric mains providing the supply, the current may be very great and do serious damage to the generators at the source through overheating if protective devices are not installed. The insulation of supply leads can be tested by measuring the current flowing through an ammeter in the

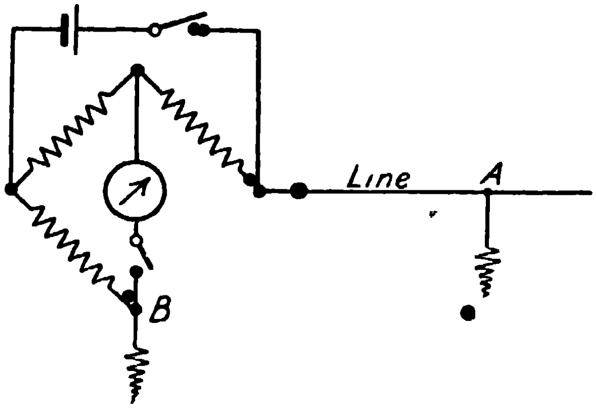
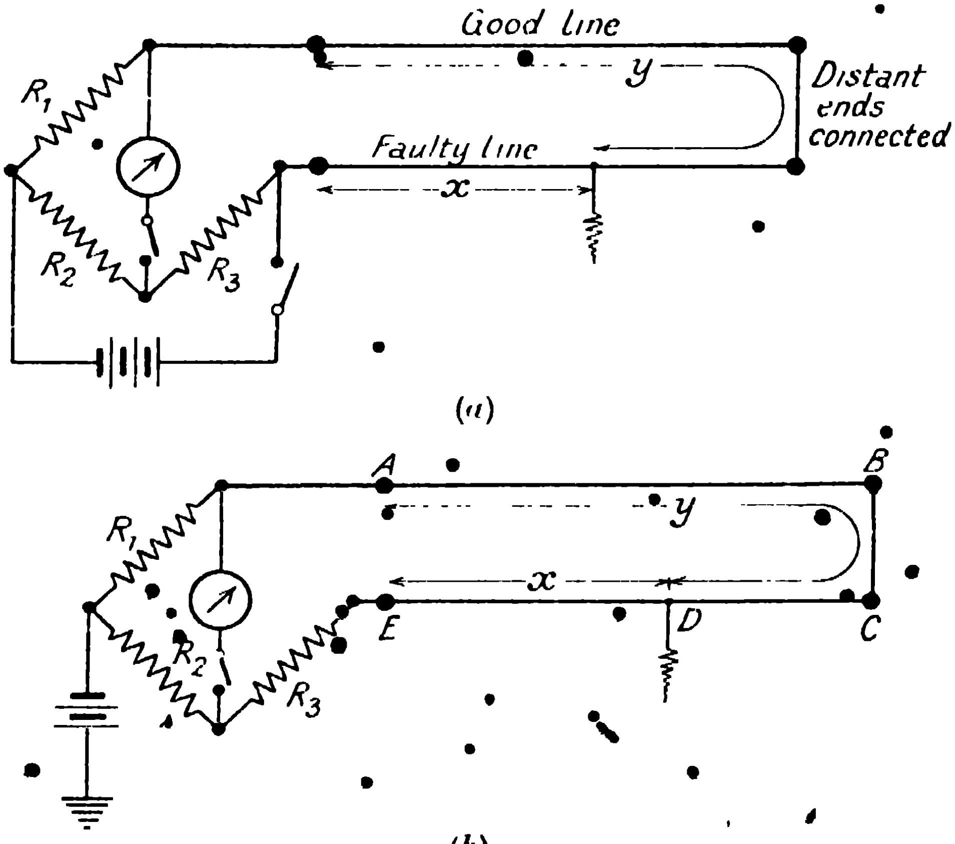


FIG. 23. LINE FAULT



(a)

(b)

FIG. 24. VARLEY LOOP TEST

position $C$, when the lead to $D$ is disconnected, and therefore no current is supposed to flow.

A simpler method is to measure the resistance between the line and earth by means of a P.O. box connected as in Fig. 23. If the line insulation is faulty, the resistance measured will give a direct indication of the fact. The earth acts as a return lead from $A$ to $B$ and the resistance measured is that of the line and fault in series. The position of a fault to earth on a line such as a telegraph or telephone line can be found when a good line is available between the same two points. The method is known as the *Varley loop test.* The resistance of the good line and the faulty line in series are first measured by a P.O. box connected as in Fig. 24 (*a*).

$R_1$, $R_2$ are the ratio arms, $R_3$ is the variable resistance in the box.

$R_1 = R_2$ generally. Let the resistance of the loop $= l$.

One terminal of the battery is disconnected from the box and connected to earth as in Fig. 24 (*b*). This is equivalent to connecting that terminal to the line at the fault.

If $y$ = resistance of $ABCD$ and $x$ = the resistance of $DE$

$$x + R_3 = y, \text{ if } R_1 = R_2$$

or

$$y - x = R_3$$

but

$$y + x = l$$

so

$$l - R_3$$

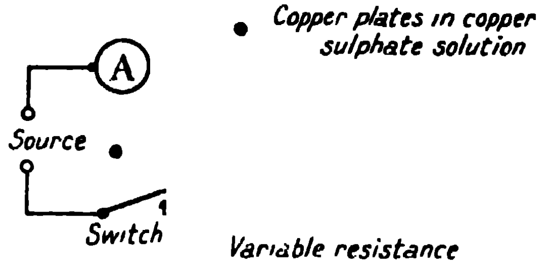
The distance to the fault can then be calculated from the resistance, which is known, of one mile of line wire. This method is very accurate for underground lines, as the temperature is fairly constant, but for overhead lines the correction of the resistance for temperature must be estimated and may not be accurate. The method assumes a definite earth fault on a well-insulated line. If the fault is a high resistance one the current supplied through it to the bridge is small and an approximate measurement only of the resistance $x$ is possible.

## CLASS EXPERIMENTS

### ON THE SUBJECT OF CHAPTER I

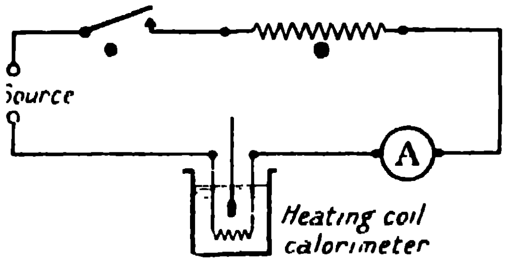
*Note.* Wherever possible these experiments should be carried out with both d.c. and a.c. sources.

1. Calibration of Ammeter by Copper Voltameter—



$$I = \frac{\text{Weight deposited}}{0{\cdot}00033 \times \text{time in sec.}}$$

2. Heating by a Current—Resistance of Conductor—



$$\frac{\text{Heat in calories } (h)}{4{\cdot}2 \times \text{time in sec. } (t)} = \text{joules per sec.}$$

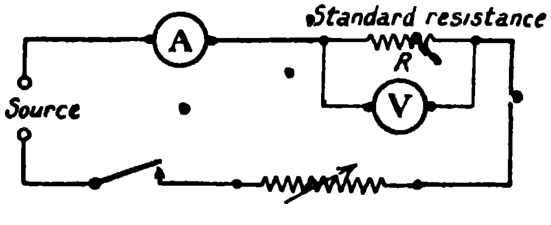
$$= \text{watts}$$

$$= I^2R$$

$$R = \frac{h}{4{\cdot}2tI^2}$$

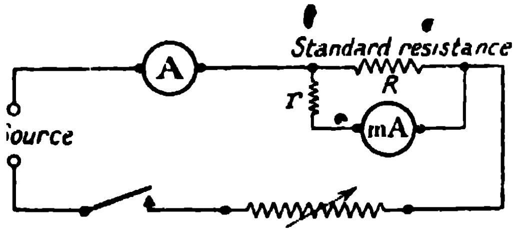
Measure $R$ at more than one value of the current.

3. Calibration of Voltmeter by Ammeter and Standard Resistance—



$$V = RI$$

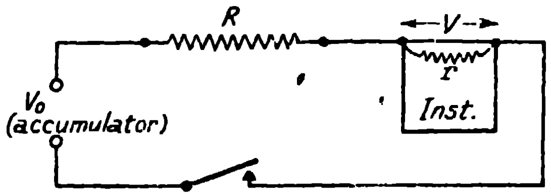
**4. Conversion of Milliammeter, etc., for Use as a Voltmeter by Series Resistance—**



Adjust $r$ so that deflection of instrument reads $V = IR$ conveniently.

Repeat for various values of $R$ and consider the effect of the relative values of $r$ and $R$. Unless $r$ is large compared with $R$ allowance must be made for the current that flows through the milliammeter, is included in the current read by A, but does not contribute to the p.d. across $R$.

**5. Measurement of Resistance of Current Instrument and Calibration for Use as a Voltmeter**



$$V = Ir,\ V_0 = I(R + r),\ R + r = \frac{V_0}{I} = \frac{\text{const.}}{I}$$

Plot $1/I$ against known values of $R$.
Intercept on axis gives $r$.

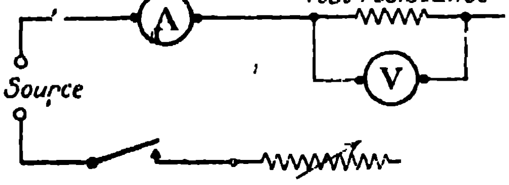
Hence, $V = Ir$

and $V_0 = I(R + r)$

Calculate $R$ to give different ranges of volts.

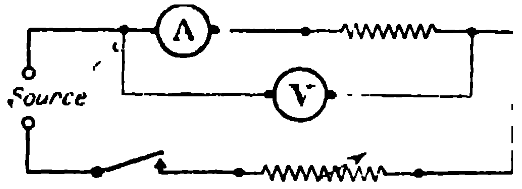
**6. Measurement of Resistance by Ammeter and Voltmeter—**

(a)



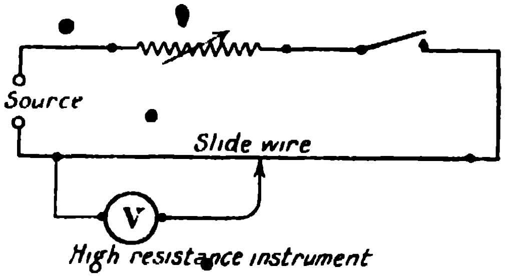
$R = \frac{V}{I}$: Find correction for current in voltmeter.

(b)



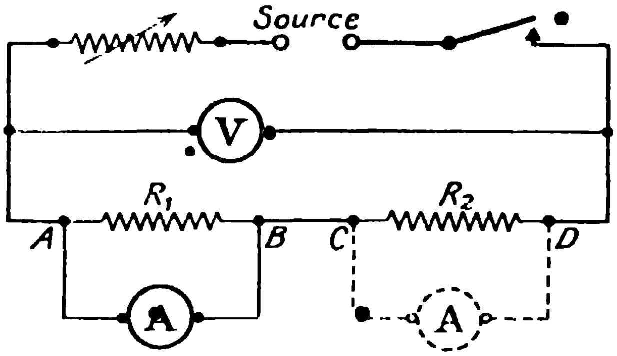
Find correction for p.d. in ammeter.

7. **Fall of Potential along a Slide Wire—**



Plot reading of voltmeter against length of slide wire.

8. Comparison of Resistances—



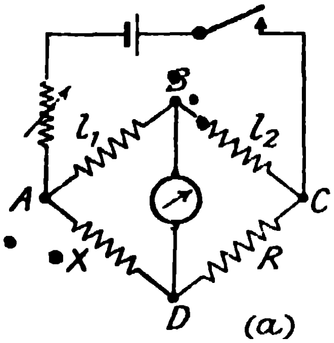
Ammeter (or voltmeter) is connected first to $A$ and $B$ and then to $C$ and $D$. $V$ is adjusted to give the same reading in each case by means of the variable rheostat.

Then with readings $I_1$ and $I_2$ on ammeter (or voltmeter)

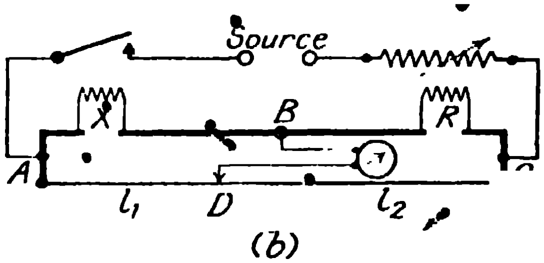
$$\frac{I_1}{I_2} = \frac{R_1}{R_2}$$

The proof is left as an exercise to the student.

9. Measurement of Resistance by Slide Wire Pattern Wheatstone Bridge –



(a)



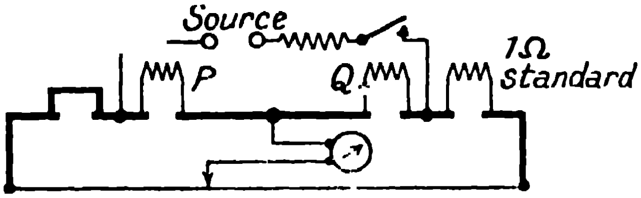
(b)

Exchange $X$ and $R$.

**10. Measurement of Resistance by P.O. Box Pattern Wheatstone Bridge—**

Connections according to scheme (*a*) in Experiment 9 above, $l_1$ $l_2$ are the ratio arms, $R$ the resistance arm in box.

**11. Measurement of the Resistance of a Slide Wire Eliminating End Corrections. Four-gap Bridge—**

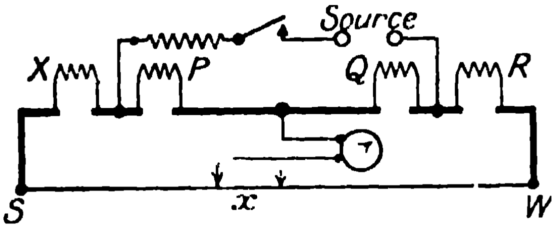


$P$ and $Q$ are equal resistances.

Balance as shown, then exchange strap $S$ and 1 ohm standard and rebalance. Slide wire between balance points has 1 Ω. resistance.

**12. Comparison (Accurate) of Two Nearly Equal Resistances.—**

Comparison of a standard with coil of the same nominal value. Carey Foster method. Four-gap bridge.



$P$ and $Q$ are equal resistances.

$X$ is the coil under test, $R$ is a standard. Balance as shown.

Exchange $X$ and $R$ and balance again.

Shift of balance a distance $n$ point gives difference between $X$ and $R$. $X - R = rn$ if $r$ = resistance of 1 cm. of $SW$.

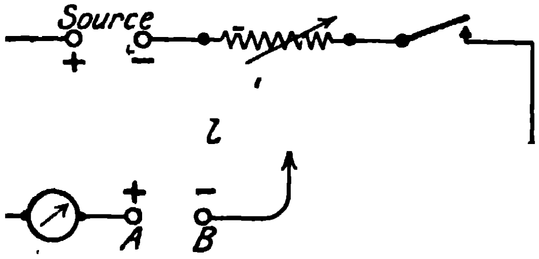
**13. Variation of Resistance of Copper with Temperature—**

A coil of about 10 ohm copper wire is heated to measured temperatures in an oil bath and its resistance measured by P.O. box or slide wire bridge. A graph of $R$ against temperature gives $R_0$ and $\alpha$. $R = R_0(1 + \alpha\tau)$.

**14. Variation of Resistance of Platinoid or Eureka Wire with Temperature—**

A coil of about 10 ohms of wire is heated in an oil bath and its change of resistance is measured by the method of Experiment 12. The results are to be shown on a graph.

**15. Potentiometer.—Comparison of p.d.'s Generated by Cells—**



The cells to be compared are connected in turn to $A$ and $B$, taking care to join + to +. The balance point is found for each on the slide wire.

$$\frac{l_1}{l_2} = \frac{V_1}{V_2}, \text{ etc.}$$

16. Potentiometer.—Variation of p.d. at Terminals of Cell with Load.

As in Experiment 15 with various resistances $R$ connected acros the terminals of the cell tested. Resistance of cell $= r$.

$$V = IR: \ I = \text{current from cell.}$$

$$V_0 = I(R + r)$$

$$\frac{r}{R} = \frac{V_0 - V}{V} = \frac{l_0 - l}{l} \text{ as } V \propto l.$$

17. Potentiometer.—Calibration of Ammeter by Standard Cell and Resistance—

P.D. on standard resistance, carrying current read on ammeter, is compared on potentiometer with p.d. from standard cell on open circuit. True amperes are given by

$$\frac{\text{p.d. on resistance}}{\text{resistance}}.$$

This is compared with the reading.

18. POTENTIOMETER.—Calibration of Voltmeter—

P.d. at terminals of voltmeter, when connected to a supply which can be varied, is compared with p.d. from standard cell or with p.d. on standard resistance carrying a current measured by standard ammeter.

19. Potentiometer.—Comparison of Resistances—

P.d.'s on the resistances are compared when they are carrying the same current.

## EXERCISES

1. Two wires of different resistances are connected in parallel to form part of a circuit through which an electric current flows. In which wire will the heat developed be the greater?

(The wire of smaller resistance.)

2. With what resistance must a galvanometer of resistance $= 189\ \Omega$. be shunted in order that only one-tenth of the current to be measured may pass through it?

(21 Ω.)

3. Two resistances, one 10 Ω. and the other 100 Ω., are connected in parallel, and the combination is connected in series with a resistance of 20 Ω. What p.d. will be required (i) to send $\frac{1}{10}$ A. through the whole circuit, (ii) to send $\frac{1}{10}$ A. through the 100 Ω. resistance.

((i) 2·91 V.; (ii) 32 V.)

4. The resistance of the armature of a generator is found to be 0·25 Ω. at 16° C. The machine is run on full load for 8 hr. when the resistance is found to be 0·28 Ω. What is the temperature of the winding at the end of the run? (45·6° C.)

5. A moving coil instrument has a resistance of 5 Ω. and requires 50 mA. for full scale deflection. How can this instrument be arranged to read (*a*) up to 10 A., (*b*) up to 10 V. (0·0251 Ω. shunt. 195 Ω. series.)

6. What size of cable is necessary to transmit 150 kW. to a consumer situated 1·5 miles from the source of supply, which is maintained at 1 200 V.? Assume a loss of 10% in the cable. (0·07 sq. in.)

7. An ammeter was connected in series with a silver voltameter and a current passed. At the end of 30 min. 205 g. of silver were deposited. The current was maintained steady and indicated 99 A. Find the percentage correction required for this reading of the ammeter. (+ 3·1%.)

8. A copper cable is carrying a current of 500 A. The cross-sectional area of the cable is 0·15 sq. in. Find the p.d. between two points in the cable separated by 1 yd. (83·75 mV.)

9. The resistances of the arms of a bridge are $AB = 10$ Ω., $BC = 5$ Ω., $CD = 10$ Ω. and $DA = 5$ Ω. The p.d. between $A$ and $C$ is 25 V. positive from $A$ to $C$. Find the p.d. between $B$ and $D$, stating which is at the higher potential, $B$ or $D$. (8⅓ V., $D$ higher.)

10. The working temperature of a 100 W., 220 V., gas-filled tungsten filament lamp is 2 600° C. What current will this lamp take from a 200 V. supply at the moment of switching on, if the room temperature be 18° C.? (5·55 A.)

11. A 5 Ω. resistor of manganin wire is to be wound (as a single-layer winding) on a circular section former which has an external diameter of 0·75 in. The diameter of the bare wire = 0·048 in.; the diameter of the covered wire = 0·060 in. How many turns will be required? (205 turns.)

12. Estimate the resistance of a heating element suitable for giving the following hot water service—

| | |
|---|---|
| Inlet temperature of water | 60° F. |
| Outlet temperature of water | 180° F. |
| Quantity required daily | 15 gal. |
| Efficiency of heating apparatus | 90 %. |
| Supply voltage | 230 V. |

What will be the weekly cost of the electrical energy required, if the energy costs 1½d. per kWh.? (216 Ω., 5s. 1½d.)

13. A factory is situated ¾ miles away from a substation and is supplied at 440 V. The efficiency of transmission, when the factory demand is 50 kW., is to be 94%. What will be the current in the cables under these conditions? (121 A.)

14. A 230 V. tungsten filament lamp has a resistance of 25·8 Ω. at 16° C. The normal working temperature of the filament is 2 700° C. Determine—

(*a*) The normal current in the lamp;
(*b*) The current taken from the supply immediately on switching on;
(*c*) The normal power rating of the lamp. (0·65 A., 8·9 A., 150 W.)

15. 10–100 W., 220 V. lamps are connected in parallel with one

another. 5–150 W., 220 V. lamps are connected in parallel with one another. The two groups are connected in series across a 440 V. supply. Determine the current in each lamp, assuming the resistances are the same as under normal running conditions. (0·39 A., 0·78 A.)

16. In a copper refinery the purification of copper is carried out by electrolysis. If there are 20 vats in series, and the cathode in each is 4 sq. ft. in area, what weight of copper is deposited per hour with a current density of 20 A. per sq. ft., the metal being deposited on both sides of the copper cathode?

(3·78 kg. per hr.)

17. An electric motor is used to work a lift. The weight of the lift loaded is 1 500 lb. and it is raised at the rate of 4 ft. per sec. The motor converts 80% of the electric energy supplied to it into mechanical work. Find the power in kilowatts required by the motor.

(10·2 kW.)

18. A block of offices is lit by 160 60 W. and 120–40 W. lamps. If the supply is at 220 V., what will be the current taken if all are switched on at the same time?

(65·5 A.)

19. The resistance of the bronze wire used for a telephone line is 44 Ω. per mile and the weight of the wire is 40 lb. per mile. What would be the resistance of the loop to a subscriber three miles from the exchange, using wire of the same material but weighing 100 lb. per mile? (105·6 Ω.)

20. It is required to supply 50 kW. to a factory 800 yd. distant from the generating station. The power is to be supplied at 400 V. at the factory. Calculate the sectional area of the copper conductor to be used, if the drop of volts in the transmission is to be 10 V. 1 000 yd. of copper bar 1 sq. in. section has a resistance at 60° F. of 0·024 Ω.

(0·24 sq. in.)

21. If the same power is to be supplied at 1 200 V., and the same fraction of the input power may be lost in transmission, what cross section of copper may be used? (0·0267 sq. in.)

22. The temperature rise due to the heating by an electric current is within a safe limit for a particular material if the power to be dissipated in the wire does not exceed $\frac{1}{10}$ W. per cm.$^2$ of cooling surface. Calculate the safe current for wires of 0·2, 0·4, 0·8 and 1·0 mm. dia., if the resistivity of the wire is 40$\mu$ Ω. per cm. Plot the results graphically.

(0·222 A., 0·628 A., 1·78 A., 2·48 A.)

23. Calculate the safe current density for the different wires in Question 7 and draw a graph of the results.

(707, 500, 353, 316 A. per cm.$^2$.)

## CHAPTER II

# CONDENSERS AND ELECTROSTATICS

**1. Condensers.** In the last chapter the conditions governing the flow of electricity in a completely conducting circuit were examined. It will now be necessary to consider the case when there is no complete conducting circuit, but the flow is barred by an insulator. No steady flow occurs when a source of steady p.d. is connected to such an interrupted circuit, but something does take place in the insulating gap, in consequence of which alternating currents can flow in the circuit.

A water analogy is helpful in understanding how this may be. In Fig. 25, *P* is a reciprocating piston and *D* is a diaphragm in a water pipe circuit completely filled with water. If *D* is made of some very elastic material such as indiarubber that can stretch and bulge out, it will be possible for *P* to move the water backwards and forwards round the circuit; and this movement of the water constitutes an alternating current. If *P* were replaced by an ordinary pump, the diaphragm at *D* would allow a small initial movement but would prevent water being forced continuously round the circuit. In this way it is possible to have a circuit in which steady direct currents are impossible, but in which alternating currents can flow.

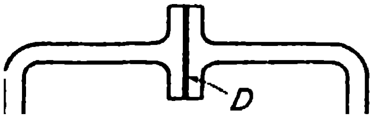


FIG. 25
WATER ANALOGY
OF CONDENSER

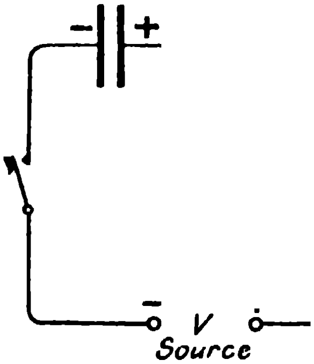


FIG. 26
CHARGING A
CONDENSER

Set up two metal plates, parallel with and facing one another at a short distance, *d* cm., apart. Connect them to the terminals of a source of steady d.c. p.d. with a sensitive current detector in one of the leads as in Fig. 26. When the switch is closed, it will be found that a current flows for a very short time and then ceases. The galvanometer pointer kicks and then swings back to zero. There is now no current in either of the leads, and therefore each plate must be at the same potential as the terminal of the source to which it is connected. The p.d. between the plates is the same as that between the terminals of the source. During the time the current was flowing a certain quantity of electricity passed from the positive terminal to its plate and an equal quantity passed from the other plate to the negative terminal; the indication on the galvanometer is the same in whichever part of the circuit it is placed. This arrangement of plates is called a *condenser*, and the quantity of electricity, *Q*, which flowed, is called the *charge* in the condenser. If the condenser is connected in a similar way to a source of higher p.d., it will be found that the impulsive indication of the galvanometer is increased in proportion to the increased p.d., showing that the charge increased with the p.d. As the current flows, then the charge carried to the condenser accumulates and raises the p.d. between the plates until it reaches a value equal to that of the source, by which time the current will have died down to nothing.

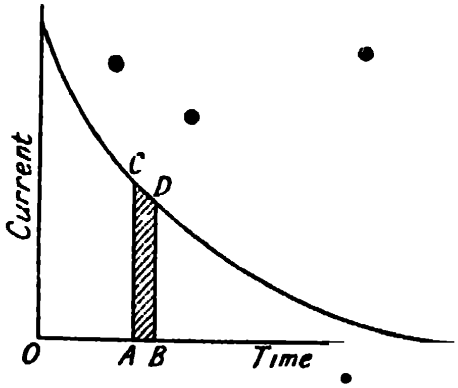


FIG. 27

CURRENT-TIME CURVE

If the graph of the current is drawn against a time base, the curve of Fig. 27 is obtained. The current at time *A* is *AC*, and a little later at time *B* it is *BD*. The charge which has flowed to the condenser during the time *AB* is the length of the interval *AB* multiplied by the average current or by the average height of the strip *ABDC*. This is exactly given by the area *ABDC*. Consequently the total charge is the total area between the curve and the base. It is useful to remember this, for it is a general rule; if the curve is drawn giving the current at each instant with time as the base, the area under the curve between any two verticals gives the charge which has been accumulated during that time.

The charge in the condenser starts at nothing and grows up to its final value. If it is known at each instant, a curve can be drawn as in Fig. 28. At time $A$, the charge is $AC$, and a little later at $B$, it is $BD$. The charge which has passed in this time is got by drawing $CE$ parallel to the base. $DE$ is the charge supplied by the source in the time $AB$; also $CE = AB$. The average current during the time $AB$ is the ratio of charge supplied to the time or the ratio $DE/CE$. This is the slope of the curve of charge. This is another important general rule—the current in any case at a particular instant is the slope at that instant of the curve which gives the charge in the condenser on a time base.

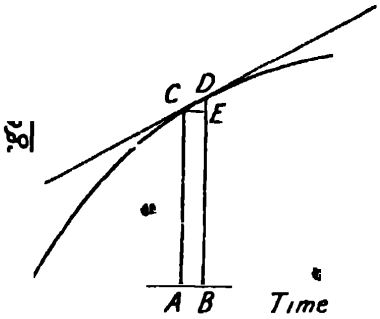


FIG. 28
CHARGE-TIME CURVE

If two or more exactly similar pairs of plates are connected in such a way as to be charged together to the same p.d., see Fig. 29, and the charging current is passed through the galvanometer, it will be found that the swing of the galvanometer pointer is proportional to the number of pairs of plates which are being charged. As it is obvious that each pair receives the same charge it follows that the throw of the pointer is proportional to the quantity of electricity which passes through it. When a galvanometer is used in this way with currents lasting for a very short time the swing of the pointer measures the total quantity of electricity, and not the current, i.e. it measures coulombs and not amperes. It is generally called a *ballistic galvanometer* when used in this way. Further, the swing of the pointer increases in proportion to the p.d. which is used to charge the condenser, and thus the conclusion can be drawn that the charge supplied to the condenser is proportional to the p.d. to which it is raised, or, expressed as an equation,

$$Q = CV$$

$C$ is a constant for a given condenser and is called its *capacitance*. The capacitance is a measure of the capacity of the condenser to hold or contain electricity. The name condenser was chosen for a similar reason, viz., the device seemed to collect or condense a noticeable charge of electricity.

The practical unit of capacitance is the *farad*; a condenser of capacitance one farad would require one coulomb to charge it to a p.d. of one volt. Or the capacitance of a condenser is the charge in coulombs required to raise its p.d. by one volt. The unit equation corresponding to

$$Q = CV$$

is
$$\text{coulombs} = \text{farads} \times \text{volts}.$$

2. **Dielectrics.** If the insulator between the plates is not air, but sulphur for example, the capacitance of the condenser is greater than the capacitance of the same pair of plates the same distance apart with air between them. It is found that the capacitance of such a condenser is the capacitance of the same condenser with air, multiplied by a factor for the particular insulator. This constant factor is called the *dielectric constant* for the insulator, and the symbol $\kappa$ (pronounced kappa, a Greek letter) is usually used to represent it. The dielectric constants of solid and liquid insulators are greater than unity, so the capacitance of a condenser is increased by using insulators other than air. This change of capacitance when the dielectric is changed means that something occurs in the space between the plates when a condenser is charged. This something is called *electric flux*. The dielectric, as it were, yields under the forces which would drive electricity across if there were a conducting path, and the amount of yielding compared with air or empty space is shown by the dielectric constant.

The dielectric constant of a substance is also called the *permittivity*, because this factor is a relative measure of how much the insulator permits the passage of the flux. As the charge flows into the condenser the flux from plate to plate increases, so that a changing flux is, as it were, a continuation of the current across the gap. The flux is constant if the charge in the condenser is constant, that is, when there is no current. The total flux between the plates is proportional to the charge.

### TABLE III

DIELECTRIC CONSTANTS

| | | | |
|---|---|---|---|
| Decane | 1·97 | Glass | 3·7–7·6 |
| Pure rubber | 2·12 | Fused quartz | 3·78 |
| Paraffin wax | 2·2–2·3 | Sulphur | 4·0 |
| Rosin | 2·5 | Gutta-percha | 4·4 |
| Paraffin oil | 2·55 | Beeswax | 4·75 |
| Vulcanized rubber | 2·7–2·95 | Mica | 5·7–7·0 |
| Shellac | 2·7–3·7 | Marble | 6·0–8·0 |
| Amber | 2·8 | Nitrobenzene | 34·0 |
| Ebonite | 2·8–3·0 | | |

Another thing which shows that there is a strain in the dielectric of a condenser is this—if the source used to charge the plates is of high enough potential difference, the insulation of the condenser may break down and become conducting, so that a current passes across. The dielectric can yield up to a point, but if the electric strain is too great, it gives way. The ability of a dielectric to withstand large p.d.'s is a different property from permittivity; it is called the *dielectric strength* of the insulation and must be considered in constructing condensers for high p.d.'s. The p.d. which a dielectric can stand without breakdown, when between a pair of parallel metal plates, is proportional to its thickness. Dielectric strength can, therefore, be measured in volts per cm.

### TABLE IV

DIELECTRIC STRENGTHS IN KILOVOLTS/CM.

| | | | |
|---|---|---|---|
| Air | 30 | Pure rubber | 330 |
| Sulphur | 33 | Paraffin wax | 600 |
| Mineral oils | 50 80 | Mica | 600–750 |
| Glass | 75-300 | Gutta-percha | 830 |
| Vulcanized rubber | 235 | Lead glass | 1 000 |
| Ebonite | 270–400 | Beeswax | 1 100 |

The volts per cm. between the plates is the work in joules done in carrying a coulomb 1 cm. on the way from one plate to the other. That is, it is the force which would act on a charge of one coulomb if it were placed between the plates, and is the force which would drive electricity along a conductor joining the plates.

A third property of a dielectric is its insulation resistance. As stated in Chapter I, no substance is a perfect insulator and consequently there is always a small leakage current flowing across between the plates when a condenser is charged. The values of the resistivities of some dielectrics in general are given in the table on page 3.

3. **Condensers in Series and Parallel.** If two condensers are connected in parallel as in Fig. 29, a larger charge will be required to produce the same potential difference between the plates. Let the charge on the first be $Q_1$, and its capacitance $C_1$, and the charge be $Q_2$ and capacitance $C_2$ for the second.

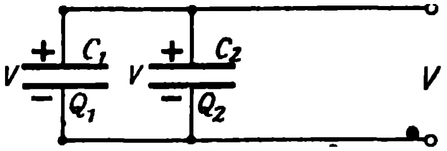


Fig. 29
Condensers in Parallel

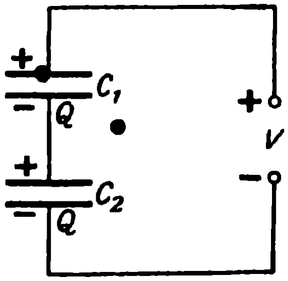


Fig. 30
Condensers in Series

Then $Q_1 = C_1V$, and $Q_2 = C_2V$.

The total charge is $Q_1 + Q_2$ and is clearly equal to

$$(C_1 + C_2)V,$$

so that the capacitance of the combination is the sum of the two component capacitances. Condensers of large capacitance can thus be obtained by joining several smaller condensers in parallel.

Condensers in Series. When two condensers are connected in series (Fig. 30), the capacitance of the combination is less than that of either. Let the two condensers have capacitance $C_1$ and $C_2$ as before, and connect them in circuit with a steady d.c. source of current giving a potential difference $V$.

A charge $Q$ flows from the + terminal of the source to the + plate of the first condenser. A charge $Q$ of the same value flows from the — plate of the first condenser to the + plate of the second condenser, and a charge $Q$ flows from the — plate of the second condenser to the — terminal of the source. There is a certain p.d., $V_1$, between the plates of the first so that $Q = C_1V_1$ and a p.d., $V_2$, between the plates of the second so that $Q = C_2V_2$. As there is no current in any of the leads once the charges are established, none of these leads has any p.d. along it. Consequently the total p.d., $V$, is equal to the sum of the p.d.'s on the two condensers.

That is, $V_1 + V_2 = V$.

If $C$ is the capacitance of the combination of condensers

$$V = \frac{Q}{C};\ V_1 = \frac{Q}{C_1};\ V_2 = \frac{Q}{C_2}$$

Therefore,
$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2}$$

or
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$$

The rule obtained from this is that when condensers are connected in series the reciprocals of the capacitances must be added to get the reciprocal of the effective capacitance. This holds however many condensers are connected in series, since the charge on each is the same and the sum of the p.d.'s is the total p.d.

$$\frac{Q}{C} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} + \text{etc.}$$

so that
$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \text{etc.}$$

4. **Energy Stored in the Condenser.** Work must be done to charge a condenser. It is easy to find out how much work is required. To increase the charge by an amount $q$ means the carrying of this charge from the negative plate round by some path to the positive plate, that is to a place where the potential is $v$ volts greater. The work is thus simply $vq$, whatever the path followed, just as the work required to lift one pound of water from a tank low down to one higher up depends only on the difference of height and not on the particular way the ascent is made. Now this addition to the charge in the condenser raises the p.d. on it, and therefore the next charge must be raised to a higher potential in taking it across to the positive plate. The p.d. $v$ is equal to $q/C$ if $q$ is the charge in the condenser.

Drawing a graph of $v$ on a base of charge $q$, a straight line $OA$ is obtained as shown in Fig. 31 because the ratio $v/q$ is constant $= 1/C$; $OL/NL = C$.

Let the charge in the condenser be $OL$ before $q$ is brought across. The p.d. is $NL$ and after $q$ has been added the p.d.

becomes $MP$ if $LM = q$. The work which has been done is $q \times$ p.d. $= LM \times$ average height of shaded strip, so the work for a small increase of charge is the area of the shaded strip. With each new addition of charge the height of the corresponding strip is increased. The total work done in charging the condenser to a final p.d. $V = BA$ and charge $Q = OB$ is the area of the triangle $OBA$. Now the area of a triangle $= \frac{1}{2}$ base $\times$ height, and therefore the work

$$= \tfrac{1}{2}OB \times BA$$

$$= \tfrac{1}{2}QV = \tfrac{1}{2}\frac{Q^2}{C} = \tfrac{1}{2}CV^2.$$

This work is stored in the condenser and can be recovered if the condenser is allowed to be discharged. As the condenser discharges from charge $= OB$ down to zero, the p.d. falls from $BA$ down to zero following the line $AO$. At any stage the charge is $OM$, for example the p.d. is $MP$ and the next charge $LM$ to leave is driven by the p.d. $MP$, and this does work $PNLM$ in pushing it out. The total work obtained from the condenser is thus equal to $OBA$ and is the same as the work spent in charging it. The energy remains stored as long as the charge is kept in the condenser.

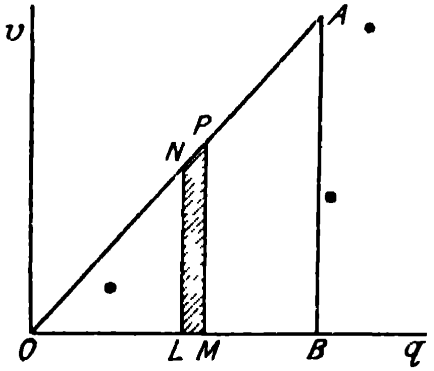


FIG. 31. ENERGY STORED IN CONDENSER

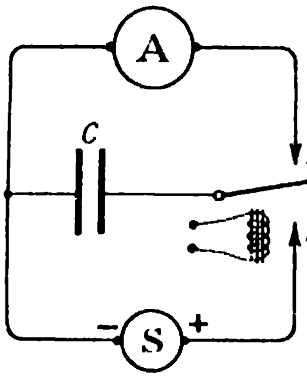


FIG. 32. MEASUREMENT OF CAPACITANCE

5. **Measurement of Capacitance.** The capacitance of a condenser can be measured by the following device. In Fig. 32 $C$ is the condenser, $S$ is a d.c. source of constant p.d., $A$ is an ammeter, and the key moves between the two contacts 1 and 2. In the position 1, the condenser is charged to the p.d. $= V$ of the source. In position 2, it is discharged through the ammeter.

The quantity which passes at each discharge is $CV$ coulombs. If these operations are repeated $n$ times per second, the quantity of electricity passing through the ammeter each second is $nCV$, i.e. the average current through the ammeter is $nCV$ amperes. The key is moved to and fro by an electromagnet fed with an alternating current from the public mains at 50 cycles per sec. This frequency is very constant. The ammeter can be put in the charging line instead of the discharging line, and it will be found to give the same reading. The current passes through the instrument in separate pulses which follow one another too rapidly for the pointer to respond to them separately, and it gives the average current, which is steady. Thus in the expression $I = nCV$, $I$, $n$ and $V$ are known, and $C$ is determined in farads from

$$C = \frac{I}{nV}$$

if $I$ is in amperes and $V$ is in volts.

The unit of capacity, the farad, is a very large capacitance, and in practice it is found that the *microfarad*, $\mu$F., which is one millionth of a farad, is a more suitable unit. Condensers whose capacitances are very small fractions of a microfarad are used in wireless circuits. The sensitiveness of the current measurer, the ammeter in the experiment just described, must be chosen so that it is suitable for measurement of the current $I = nCV$. For example, if $n = 50$, and $V = 200$ volts from a h.t. battery, $nV = 10\,000$ and 1 microfarad will give a current of 10 milliamperes. If a wireless variable air condenser of maximum capacitance 0·0005 $\mu$F. is tested, a mirror galvanometer of moderate sensitivity will be required, as the current will be 5 microamperes under the same conditions. The variation of the capacitance of an air condenser with the position of the vanes, or the increase of the capacitance when the condenser is immersed in an insulating oil, can be readily measured in this way.

6. **Condensers in A.C. Circuits.** The capacitance of a condenser can be measured, as has been seen, by arranging a vibrating switch to charge and discharge the condenser from a source of steady d.c. p.d. As the charging current and the discharging current flow along different conductors, they can be measured by using an ammeter suitable for measuring

direct current. The pointer of this instrument sets at the average value of the current pulses, which are always in the same direction. Fig. 33 gives a graph of the pulses of current charging the condenser. The average current is shown by the dotted line. The area under this line is the sum of the areas shaded under the "saw tooth" curve. The reading gives the steady current which would deliver the same number of coulombs per sec. as the pulsating current.

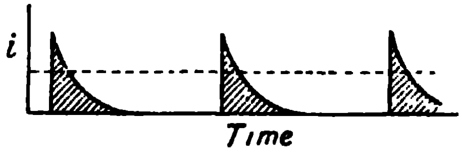


FIG. 33. CURRENT IN GALVANOMETER

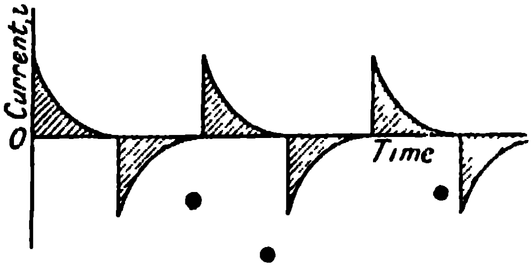


FIG. 34. CURRENT IN CONDENSER LEAD

Now, if the ammeter is put in a part of the circuit where both the charging and discharging currents flow, that is, between the condenser and the key in Fig. 32, there will be no deflection of the pointer, for it will be urged equally in each direction alternately, and as it cannot respond rapidly enough, it will remain at rest. The current pulses in this part of the circuit are shown in Fig. 34. As the charge that enters the condenser is equal to the charge that leaves it in any long interval of time, the area above the zero line is equal to the area below it. This is an alternating current and the direct current instrument reads zero. The p.d. at the terminals of the condenser rises as the charge increases and reaches a steady value equal to that of the source until the switch is changed over and discharge begins. The p.d. then falls to zero in the same way as the current decline shown in Fig. 27. The curve of rise and fall of p.d. is shown in Fig. 35.

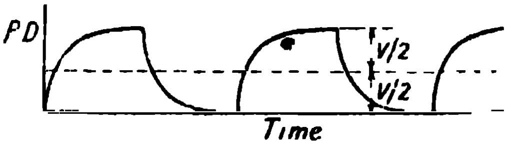


FIG. 35
P.D. ACROSS CONDENSER

The p.d. thus rises and falls about an average value equal to half the maximum p.d. The changes in this value can be simply described as—

1. A steady p.d. = $V/2$.

2. Added to this an alternating p.d. which changes from a maximum of + ($V/2$) to a minimum of – ($V/2$); the change from either value to the other taking place in the same manner.

The curve in Fig. 36 shows this alternating p.d. separated from the accompanying steady p.d.

It is possible to have in the same circuit a direct current with a steady p.d. and an alternating current with the corresponding alternating p.d. In the case just examined, the behaviour of a condenser in a circuit is shown. The steady p.d. and the alternating p.d. together cause only an alternating current; the steady p.d. sends no current through. The condenser passes alternating current, but prevents any direct current passing round the circuit.

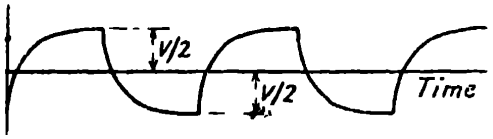


Fig. 36
A.C. P.D. Across Condenser

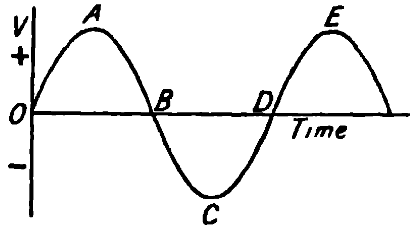


Fig. 37
Alternating Voltage

Referring to Figs. 27 and 33, it is seen that if the current in a circuit is plotted on a time base, the area under the curve is the amount of charge which has passed. As the charge which flows into a condenser raises the p.d. in proportion to the charge (in fact, the p.d. is the charge divided by the capacitance of the condenser), the p.d. on the condenser into which the current is flowing is equal to the area under the curve divided by the capacitance.

The behaviour of a condenser in a circuit with an alternating p.d. is interesting and very important. The type of alternation aimed at in a generator of alternating voltage is a smooth and regular rise and fall in each direction as in Fig. 37. This is repeated regularly, so that the maximum p.d. in the positive direction at *A* and *E*, and so on, is the same and is equal to the maximum p.d. in the negative direction at *C* and so on. The p.d. changes gradually without any sudden jump in value or in the rate of change as shown by the slope of the curve. In fact, the ideal alternation can be seen by looking at a point on the rim of a wheel, turning at a uniform rate, from a point of view in the plane of the wheel (the wheel must be imagined to be transparent). The point $P$ (Fig. 38) rises and falls in a regular way. Looked at from the side, $P$ is seen going round a circle with uniform speed. The angle $OP$ makes with the starting line $OA$ increases steadily. If the wheel turns through an angle $\omega$ in each second, then in $t$ seconds from the start it has turned through an angle equal to $\omega t$, so that the angle $AOP = \omega t$. The height of $P$ above the direct line of view is $PM$ and when seen edgewise $M$ and $A$ fall together, so that $M$ appears to be at rest and $P$ seems to rise and fall along a straight line. If equal distances round the circumference of the wheel are marked with calipers, it is known that the circumference of the wheel moves through each of these spaces in the same time. Measure the height of $PM$ when $P$ has moved through each space. That is, measure the perpendicular distance of each point of the circumference from $OA$ and draw a graph of these heights on a base of time.

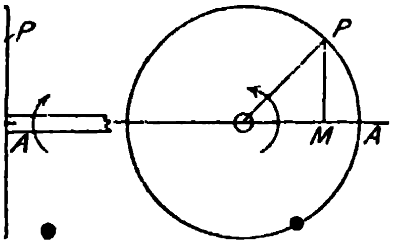


FIG. 38
NATURE OF ALTERNATION

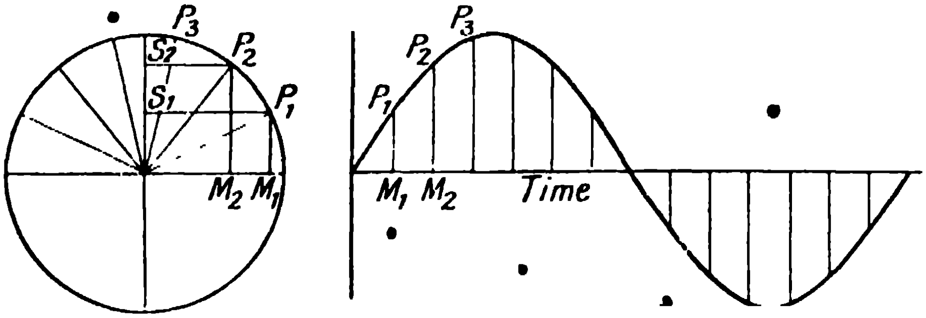


FIG. 39. CONSTRUCTION OF WAVE FORM *

A curve is obtained like that shown in Fig. 39.

It shows how the p.d. rises and falls smoothly with time. If such an alternating p.d. is applied to the ends of a wire, the current rises and falls and reverses with the rise and fall and reversal of the p.d. The current at any instant is $V/R$ and the graph of the current can be drawn directly by dividing each ordinate $PM$ of the curve by the value of the resistance $R$.

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* This is called a *sine wave*.

It is clear that a curve of the same shape will be obtained, as the p.d. curve has merely been changed in scale according to the value of $R$. The current rises and falls with the p.d. exactly in step.

Now, let such an alternating p.d. be established across a condenser whose capacitance is $C$. Current will flow in and out of the condenser in such a way that the charge in the condenser is at any instant equal to $Cv$, where $v$ is the p.d. at that instant. That is, the charge in the condenser at each instant can be shown by a similar curve to the p.d. curve in which each ordinate equals $PM$ multiplied by the value of the capacitance $C$.

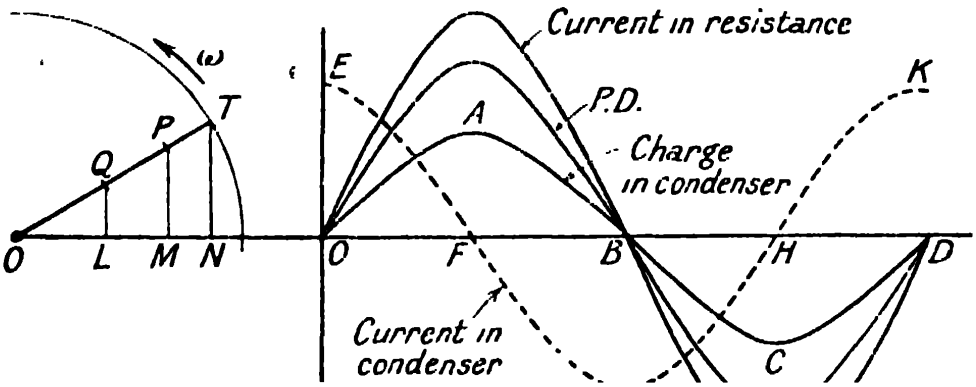

Fig. 40. Charge, P.D., and Current Curves

To sum up, Fig. 40, if the alternating p.d. has a maximum value $V$, its variation can be represented by the help of a line $OP$ of length $V$ rotating round $O$, so as to complete one whole rotation in the time of one complete alternation of the volts. The p.d. at any instant ($= v$ volts) is equal to the length of the line $PM$. The current at that instant that would flow in a resistance $R$ ohms to which the p.d. is applied would be equal to $TN$, if $OT = V/R = I$, where $I$ is the maximum value of the current. $TN$ is therefore equal to $PM/R = v/R$ for every position of the line $OP$ as it is rotated. The charge at any instant in a condenser with its terminals connected to the supply of p.d. would be obtained from a length $OQ$ along $OP$ equal to $C.V = C.OP = Q$ where $Q$ is the maximum value of the charge. The charge at this instant is $QL$, for $QL = C.PM = \mathrm{C}.v$. The curves showing the current $I$ in a resistance $R$ ohms and the charge $Q$ in a condenser of capacitance $C$ farads are shown in Fig. 40.

A knowledge of the current flowing into and out of the condenser is generally of more importance than a knowledge of the charge. From $O$ to $A$ the charge is increasing, rapidly at first but at a diminishing rate until at $A$ the charge is a maximum and is not changing.

During this time the current, which, as has been seen, is the rate of change of charge, i.e. the slope of the curve $OA$, falls from $OE$ at $O$ where the condenser is being charged most rapidly, to zero at $F$, where momentarily the charge in the condenser is not changing. The current curve $EF$ is obtained by plotting for every instant the slope of the charge curve $OA$. The remainder of the current curve $FGHK$ is obtained from the charge curve $ABCD$ in the same way. From $A$ to $B$ the condenser is being discharged; slowly at first and at the maximum rate at $B$ where the charge is zero. Current during this period is flowing out of the condenser, i.e. in the reverse or negative direction. The current curve $FG$ is therefore drawn below the axis, and the current is zero at $F$ and reaches a negative maximum at $G$. From $B$ to $D$ the curves $BCD$ and $GHK$ are $OAB$ and $EFG$ repeated with the sign everywhere reversed. During this time the condenser is charged in the opposite direction (what was previously the positive terminal is now the negative) and discharged; the charge is a maximum at $C$. At $D$ the conditions are once more exactly the same as at $O$, and the whole alternation or *cycle*, as it is called, is repeated over and over again as long as the supply is maintained.

The shape of the current curve is seen to be similar to that of the others, but it does not reach its maximum or cross the base line at the same time as they do. The various stages of the current alternation occur one-quarter of a cycle earlier than the same stages of the charge or of the p.d. alternations. The current wave is out of step with the p.d. wave by one-quarter of a cycle and is in advance. The current is said to *lead* the p.d. by one-quarter cycle. The maximum value of the current is equal to the maximum rate of change of charge and this is equal to the capacitance multiplied by the maximum rate of change of p.d. The p.d. is rising fastest when the point $P$ on the circle is passing through the base line at $A$ in

Fig. 38. The rate of rise at that stage is easily calculated, as it is simply the upward velocity of $P$, and that is the same as the speed of $P$ round the circle. If $P$ goes round $n$ times per second, since each time round it moves through a distance equal to the circumference, which is $2\pi \times OP$, in one second it moves through $n \times 2\pi \times OP = 2\pi n V$, and this is the speed of $P$. The greatest rise of p.d. is therefore $2\pi n V$ in volts per sec.

The greatest value of the current is then

$$C \times 2\pi n V = 2\pi n C V = \omega C V,$$

writing, as is usual and as has been done on page 53, $\omega$ for $2\pi n$.

A line from which the current curve can be constructed in the same way as the p.d. and charge curves can now be found. It is drawn as $OF$ equal to $\omega C V = \omega C \times OP$ at right angles to $OP$ as in Fig. 41. $OP$ represents the alternating p.d. as it rotates and $OF$ will represent the current charging the condenser in the same way. At any instant such as that corresponding to the positions of $OP$ and $OF$ as in the figure,

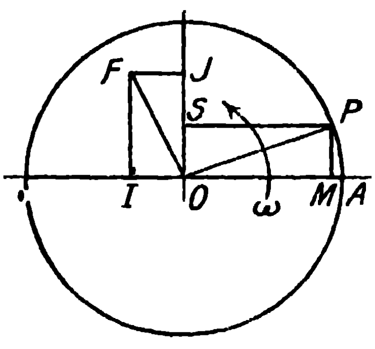

Fig. 41
Vector Diagram

$$PM = \text{p.d. at that instant},$$
$$FI = \text{current at that instant}.$$

$FI$ rises and falls and reverses one-quarter of a turn in front of $PM$ as the current does and it reaches the correct maximum value, $\omega C V$.

Curves like those in Fig. 40 can be drawn to show the values at various times of any alternating quantity. In such a diagram the essential facts are—

(1) the maximum value,
(2) the number of cycles per sec.,
(3) the times at which maximum and zero values are reached.

Any method which gives these three things will tell all that is necessary, because, by the help of the circle with a rotating radius, the curve showing the quantity on a time base can then be drawn. In drawing this curve, it is necessary, first of all, to draw a circle with centre $O$ and radius equal in length to the maximum value. This shows fact (1).

Next, write on the circle the value of $\omega = 2\pi n$, fact (2).

Then a radius of the circle is drawn showing the position of the rotating line at some instant from which the time can be reckoned, fact (3).

If it is wished to draw the curves for several related alternating quantities, a radius for each can be drawn with its proper length (1) and direction (3) and the value of $\omega$ which is the same for all can be noted. Then the circles can be drawn, and the curves. Generally, it is necessary to go no further than to draw the lines in their proper lengths and directions, for they show all that is needed, and they can be used for calculation and study without the trouble of drawing the curves.

For example, a condenser of 2 $\mu$F. capacity is connected to a source of p.d. of 20 volts maximum value and 120 cycles per sec. What is the current?

Draw a line $OP = 20$ volts to represent the alternating p.d. Draw a line $OF$ at right angles to $OP$ and mark off on it a length $\omega CV = 2\pi \times 120 \times 20 \times 2 \times 10^{-6} = 9{\cdot}6 \times 10^{-3}$ amperes. This represents the current required. The maximum value is 9·6 milliamperes and is $\frac{1}{4}$ cycle or 90° in advance of the p.d.

The relation between the current through a condenser and the applied p.d. may be written $I = j\omega CV$. The letter $j$ is used to mean a right-angle turn of the line $OF$ forward, i.e. counterclockwise from the direction of $OP$.

If the magnitude of the current only is considered and not the position of the current-time curve in relation to the p.d.-time curve, it is sufficient to write $I = \omega CV$. Comparing this with Ohm's law for a circuit with resistance, $I = V/R$, it is seen that the condenser opposes the passage of the current to the extent of $1/\omega C$ ohms, since $I = \omega CV$ can be written

$$I = \frac{V}{\dfrac{1}{\omega C}}.$$

$1/\omega C$ is called the *reactance* of the condenser at the frequency $f = n = \omega/2\pi$. It is not the same thing as a resistance, although it is measured in ohms, for the current is not in step with the p.d.

Resistance and reactance behave differently in a circuit though each opposes the passage of the current in its own way. The important difference between them is this; resistance in a circuit causes a wastage of energy $= RI^2$ joules per sec., and this energy is converted to heat and is not recoverable by the circuit. Reactance causes no wastage of energy at all, the condenser takes in energy as it charges up and feeds that energy back to the circuit undiminished as it discharges. The two may be compared with friction and elasticity in mechanics. Energy spent in overcoming friction in any movement is converted into heat and does not help the mechanism in any way. Energy spent in compressing a spring is returned by the spring when it expands. The energy is in the meantime stored in the compressed spring.

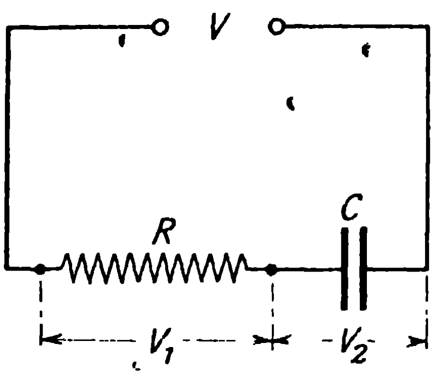

FIG. 42. CAPACITANCE AND RESISTANCE IN SERIES

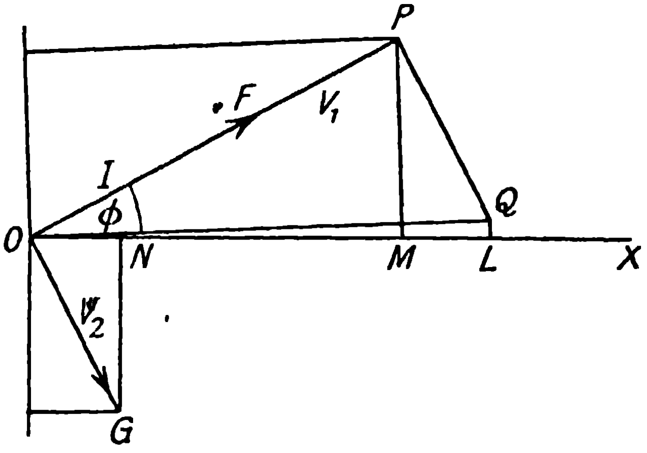

FIG. 43. VECTOR DIAGRAM

The effect of a condenser in series with a resistance when an alternating p.d. is supplied to the circuit can be found in a way similar to the last example. Let the resistance (Fig. 42) be $R$ ohms, the capacitance $C$ farads, and the voltage of supply be $V$ volts maximum at a frequency $\nu = \omega/2\pi$ cycles per sec. Let the current in the circuit be $I$ amperes, maximum, and the maximum p.d. across $R$ be $V_1$ and across $C$ be $V_2$. Then $V_1 = RI$ and $V_2 = I/\omega C$ with $I$ and $V_1$ leading $V_2$ by a quarter cycle.

Let the radius representing the current at a particular instant be $OF$ (Fig. 43).

Then $$V_1 = R \times OF = OP$$

and $$V_2 = \frac{1}{\omega C} \times OF = OG$$

where $OG$ is drawn at right angles to $OF$.

The value of the total p.d. at any instant is the sum of the heights of $P$ and $G$ above $OX$, the base. As $GN$ is below $OX$ the p.d. on the condenser is as it happens opposite to that on the resistance at this particular instant. If $PQ$ is drawn parallel and equal to $OG$, the height $QL$ of $Q$ above $OX$ is the total p.d. at the instant for which the diagram is drawn. As $OP$ and $OG$ rotate together and keep fixed in size the triangle $OPQ$ also rotates without alteration, and at each instant the height $QL$ of $Q$ above $OX$ gives the p.d. total. $OQ$ must therefore be the value of the maximum p.d. $= V$, and must be the line from which the total p.d.-time curve is drawn.

As $OPQ$ is a right-angled triangle

$$OQ^2 = OP^2 + PQ^2$$

i.e. $$V^2 = R^2I^2 + \frac{I^2}{\omega^2C^2} = I^2\left[R^2 + \frac{1}{\omega^2C^2}\right]$$

thus $$V = I\sqrt{\left[R^2 + \frac{1}{\omega^2C^2}\right]} = IZ$$

where $Z$ is a constant of the circuit at the frequency considered.

The effective or total opposition of the resistance and capacitance in series is not got by adding their oppositions by simple arithmetic, but by geometrical addition.

$Z = \sqrt{\left[R^2 + \frac{1}{\omega^2C^2}\right]}$ is called the *impedance* of the circuit. The impedance is made up of two parts, the resistance $R$ and the reactance $1/\omega C$. These are added by drawing them as the sides of a right-angled triangle, when the hypotenuse is their sum.

The angle $QOP$ is of importance also, as it is necessary to know this angle as well as the lengths $OP$ and $OQ$ in order to draw the two lines $OQ$ and $OP$ from which the p.d. and current-time curves of the circuit may be determined. It is called the *phase* angle between current and p.d. and can readily be measured from the diagram.

The lines that represent the current and p.d. are called *vectors* as they are added in the same way (compare the parallelogram and polygon of forces) as vectors are added. They have magnitude, direction and sense, but they do not represent vector quantities and are not true vectors.

The case of a condenser $C$ in parallel with a resistance $R$ across a supply of p.d. $V$ is readily dealt with in the same way. Let $I$ be the total current (Fig. 44) from the supply, $I_1$ the current in the resistance, $I_2$ the current in the condenser, and $V$ the p.d. of the supply which is common to both the resistance and the condenser. In the line or vector diagram, $OV = V$, $OI_1 = V/R$ drawn along $OV$, and $OI_2 = \omega CV$ drawn at right angles (leading) to $OV$. $OI_1$ and $OI_2$ added vectorially (by the parallelogram law) give $OI$ as the total current $I$.

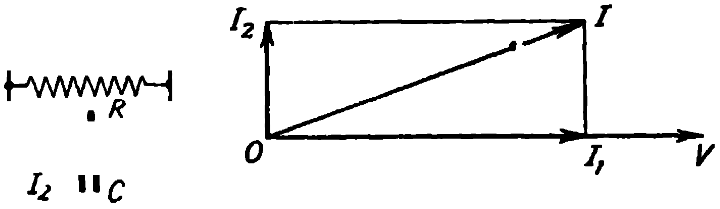

FIG. 44. CAPACITANCE AND RESISTANCE IN PARALLEL

Clearly

$$OI^2 = OI_1^2 + OI_2^2$$

i.e.

$$I^2 = \frac{V^2}{R^2} + \omega^2C^2V^2$$

$$I = V\sqrt{\left[\frac{1}{R^2} + \omega^2C^2\right]}$$

If $Z$ is the impedance of the parallel circuit, since

$$V = IZ$$

$$Z = \frac{1}{\sqrt{\left[\frac{1}{R^2} + \omega^2C^2\right]}}$$

$$= \frac{R}{\sqrt{(1 + \omega^2C^2R^2)}}$$

It should be noted that if $R$ is very large $\omega^2C^2R^2$ is very large compared with 1 and $Z = 1/\omega C$ very nearly; the condenser only is of importance. If $C$ is very small, $\omega^2C^2R^2$ is very small compared with 1 and $Z = R$; the resistance only is of importance.

As has been mentioned before, a condenser in an a.c. circuit absorbs no power; all the power taken from the supply is absorbed in the resistance parts of the circuit. The power in the resistance at any instant is $i^2R$, where $i$ is the current in the resistance at that instant. The power absorbed over a period of time is accordingly the average value of $i^2R$, or the mean value of the square of the current multiplied by the resistance. If therefore the root mean square of the current is $I_{r.m.s.}$, the power is simply $I^2_{r.m.s.} \times R$. It has been shown in the first chapter that generally when alternating currents are in question they are measured by their r.m.s. values; the square of these values multiplied by the resistance gives the power.

Alternating current supplies approximate very closely to the type shown in Fig. 39 and in this case it is readily shown by elementary trigonometry* that the mean square value of the current is one-half the square of the maximum value, i.e. $I_{r.m.s.} = I_{max}/\sqrt{2}$.

The power is also given by the mean value of $vi$, where $v$ is the p.d. across the resistance at any instant, and $i$ is the current in it at that instant. Since

$$v = iR$$

and $$V_{r.m.s.} = I_{r.m.s.}\,R$$

the power
$$\begin{aligned} P &= I^2_{r.m.s.}\,R \\ &= (I_{r.m.s.} \times R) \times I_{r.m.s.} \\ &= V_{r.m.s.}\,I_{r.m.s.} \end{aligned}$$

If r.m.s. values are used the power spent in a resistance is

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[*] The type of curve of Fig. 39 is expressed by $i = I_{max} \sin \omega t$. The value of $i^2$ is accordingly given by

$$\begin{aligned} i^2 &= I^2_{max} \sin^2 \omega t \\ &= I^2_{max}(\tfrac{1}{2} - \tfrac{1}{2}\cos 2\omega t) \end{aligned}$$

The mean value of $\cos 2\omega t$ over a cycle is zero. Therefore the mean value of $i^2$ is $\frac{1}{2}I^2_{max}$.

given by the product of the p.d. across it and the current through it. If, however, the maximum values are given, since

$$V_{max} = (\sqrt{2})V_{r.m.s.}$$

and $$I_{max} = (\sqrt{2})I_{r.m.s.}$$

the power $$P = \frac{V_{max}}{\sqrt{2}} \cdot \frac{I_{max}}{\sqrt{2}}$$
$$= \tfrac{1}{2}V_{max}\ I_{max}$$

It very often happens that the p.d. across, and the current through, the resistance parts of an a.c. circuit are not separately known, but only the p.d. across the whole circuit, the current through the circuit and the phase angle between them. Referring, for instance, to Fig. 43, $V = OQ$ is known and $I = OF$, and the angle $POQ = \phi$ between them. It has been seen above that the power in the circuit (using r.m.s. values for p.d. and current) is $I \times V_1$, i.e. $OF \times OP$. If $OP$ is not known, it can be found by drawing $QP$ perpendicular to $OP$, or in other words, it is the projection of the p.d. vector on the current vector. This rule holds in all circuits, however complicated. The power is given by the product of the length of the current vector and the projection upon it of the p.d. vector (or what is the same thing, the product of the length of the p.d. vector and the projection upon it of the current vector), the lengths of the vectors being the r.m.s. values of the quantities they represent.

This rule can be written

$$\text{power} = VI \times \text{a constant}$$

where the constant is the ratio of $OP$ to $OL$ and is known as the *power factor* of the circuit.*

The charges on the plates of a condenser have been discussed, but these charges are not the only ones present. The leads to the plates and any conductors connected to them are charged; and only differ from the condenser plates in having a smaller area and in being a great distance apart. There is capacitance associated with all parts of the circuit, but as the capacitance between two conductors is proportional to the ratio $\frac{\text{area}}{\text{distance apart}}$ the capacitance of the leads is in general too small to produce any noticeable effect. Two exceptions must be made. The first is the case of the mains from an a.c. generating station, particularly if they are long. The capacitance of the leads may allow an appreciable current to pass. The other case is when the frequency of alternation is high. The reactance of a capacitance $C$ is $1/\omega C$, and this is small when $\omega = (2\pi \times \text{frequency})$ is great. At wireless frequencies of a million per second and thereabouts a very small capacitance may allow a relatively large current to flow.

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* The power factor is $\cos\phi$ in the case considered.

7. **Electric Charges.** It is necessary to get some idea of what really occurs between the plates of a charged condenser, and this can be done by examining the behaviour of electric charges, how they are produced, and how they act on one another.

Electric charges are concerned in a particular way in the conduction of electricity by solutions (*electrolysis*). Each atom of a metal is accompanied by a definite charge of electricity. It holds to it, and these charges only travel through the liquid, and into or out of the electrodes, with the atoms of the metal. This electric charge appears to be a part of the structure of the metal atom and cannot be removed from it. This is made clearer by every discovery in the study of electricity since the time of Faraday. Again, when two bodies are rubbed together, or even when they are merely placed in contact and separated, charges of electricity are often found. If care is taken in the insulation of the bodies, charges are found to be produced in every case. Some wear occurs whenever two bodies touch or rub, even though it be extremely small, and along with the removal of material at the surface which constitutes wear there is removal of electric charge. When this method of producing charges is studied, it is found that there are two kinds of electric charge. These have exactly opposite properties, and when present in equal amounts they neutralize one another's influence. These two are called *positive* and *negative* respectively to express this opposition, but negative charges are as actual and real as the other sort—positive charges. Actually the charges taking part in the flow of electricity in a wire are of the so-called negative kind, but both positive and negative charges are active in electrolysis.

If a glass rod is rubbed with silk or fur, it is found that it will attract light pieces of material, such as feathers or scraps of paper. It is charged or electrified. Now suspend it in a paper stirrup by a silk thread so that it can turn round horizontally. Rub another glass rod and bring it near to one end of the first. The suspended rod swings away; it is repelled. Now rub an ebonite rod with flannel and bring it near an end of the suspended glass rod. The glass rod is attracted. If an electrified ebonite rod is hung up, it will be attracted by an electrified glass rod and repelled by an electrified ebonite rod. The charges produced on the glass and on the ebonite are clearly different; that on the glass is positive and that on the ebonite is negative. It appears then that similar charges repel and dissimilar charges attract one another. Care must be taken that the rods and the rubber are as dry as possible, or the charges produced by rubbing may leak away to earth through a film of moisture on the surface of the rods and the body. The nature of the charge, whether it is positive or negative, produced by friction is liable to vary with pressure or other circumstances. If the surface of a glass rod is roughened with emery, it is generally found to get a charge when rubbed with silk opposite to that produced by rubbing a polished glass rod with silk. Two rods of the same material rubbed together get opposite charges. The amount of charge obtained and its nature depend on the particular way the surfaces are disturbed in the rubbing. Equal charges of positive and negative electricity are always obtained.

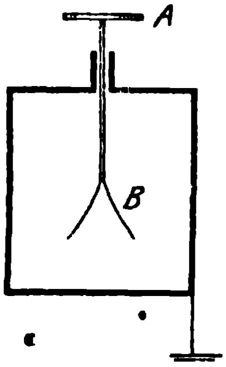


Fig. 45. Gold Leaf Electroscope

For the detection of charges the gold leaf electroscope is the most convenient instrument. This consists, Fig. 45, simply of a metal rod with two narrow strips of gold leaf *B* fastened side by side at its lower end. The rod is supported by and passes through an insulating plug in the top of a metal box. The box has glass sides to allow the leaves to be seen. The rod generally ends in an horizontal metal disc *A* at the top. If there is a charge on the electroscope the leaves repel one another, as they have charges of the same kind. If a body charged with the same kind of electricity is brought near the plate at the top, the repulsion of like charges for one another drives the charge down into the leaves to a greater extent, and their charges being increased, the leaves repel one another more strongly, and they diverge further. If a charge of opposite sign is brought near the plate, the attraction of opposite charges for one another draws the charge up to the plate from the leaves and they fall closer together. This is a simple means of finding what is the sign of the charge on a body and of showing the opposite behaviour of positive and negative electricity.

Now consider what happens when a charge (say positive in a body *C*) is brought near to the disc *A* when there is no charge on the electroscope. The positive charge on *C* exerts a force on positive charges near it, repelling them; it also exerts a force on negative charges, attracting them. There is, therefore, a force along the conductor *AB* urging positive electricity from the plate to the leaves. This force gives a p.d. along the rod making *A* a higher potential than *B*. By Ohm's law a current flows from *A* to *B* carrying a charge to the leaves. That is to say, a positive charge is produced on the leaves. Where does the electricity come from? The only explanation which will fit this case, and all other cases as well, is that in the original uncharged electroscope rod and leaves there was a large amount of both kinds of electricity, and that these two were equal in amount and so exactly neutralized one another's effects. The same effects of a current would be produced if the positive charges moved down the rod as if the negative charges moved up it. In both cases there would be positive electricity surplus in the leaves and negative electricity surplus at the top. It cannot be told from this experiment which electricity moves, or whether both move. The movement of electricity—the current—ceases when the whole conductor, consisting of plate, rod, and leaves, is at the same potential, i.e. when there is no energy available to move electricity about on them. Such is the case when the force on any part of the positive charge at the bottom, *B*, of the rod due (i) to repulsion of the rest of the positive charge near it at *B*, and (ii) to the attraction of the negative charge at the other end, *A*, of the rod just balance the repulsion of the positive charge at *C*. When the charge *C* is taken away the forces, repulsion of like and attraction of unlike charges, cause currents to flow in the rod until again a balance is reached.

The two charges mingle until there is no surplus charge in any part.

If a piece of silk or other suitable rubbing material is wrapped round a rod of glass or ebonite, etc., and the rod is turned round in the wrapping and brought, still wrapped, near an electroscope, no effect on the leaves can be seen. But if the wrapping is taken off and either the wrapping or the rod is brought near the electroscope, the leaves will separate equally. This shows that both have been charged by the rubbing and that the charges are equal. Electricity is never generated by any operation performed, but the two electricities which are present all the time can be separated by various means and drawn apart. To draw them apart means doing work—spending energy—and any agency which spends energy separating the two kinds of electricity is called an *electromotive force*. The amount of the electromotive force is measured by the p.d. in volts it can cause, that is, by the work done in separating one coulomb of positive from its partner coulomb of negative electricity. The charges which have been separated will do the same amount of work in coming together as has been spent in separating them, for, at each stage of their coming together, the force pulling them together is the same as it was at that stage while they were being separated. All electric sources are means by which energy is spent in separating the two kinds of charge. In cells the energy comes from chemical combinations, and in the dynamo and alternator it is supplied by the engine which drives the so-called generator. In the production of charges by rubbing, the work of separating the charges is a part of the work done against friction.

8. **Electric Flux.** How can an electric charge act on another charge by a force across empty space? The reason is that the charge causes a strain in all the space round it, and another charge when put in this space is affected by the strain and is drawn towards the first charge if it is of opposite kind, and repelled if it is of the same kind. This strain is the flux that was spoken about when discussing the condenser.

The following simple experiment is of great value in obtaining a mental picture of what happens in *electric fields*, as the spaces in which electric flux exists is frequently called.

Two strips of tin-foil are pasted on to a glass plate parallel with each other and a centimetre or two apart, and small gypsum crystals are scattered all over the plate. Then the strips are connected to the terminals of a Wimshurst or other "influence" machine which will establish between them a p.d. of 3 000 volts or more, and the glass plate is gently tapped. Between the tin-foil strips the crystals will arrange themselves in parallel lines at right-angles to the tin-foil, and at the ends of the strips there will be some bulging lines, but on the rest of the plate there will be no alteration in the haphazard arrangement. The general arrangement is shown in Fig. 46 (*a*), where only those crystals which have arranged themselves in a particular manner are shown.

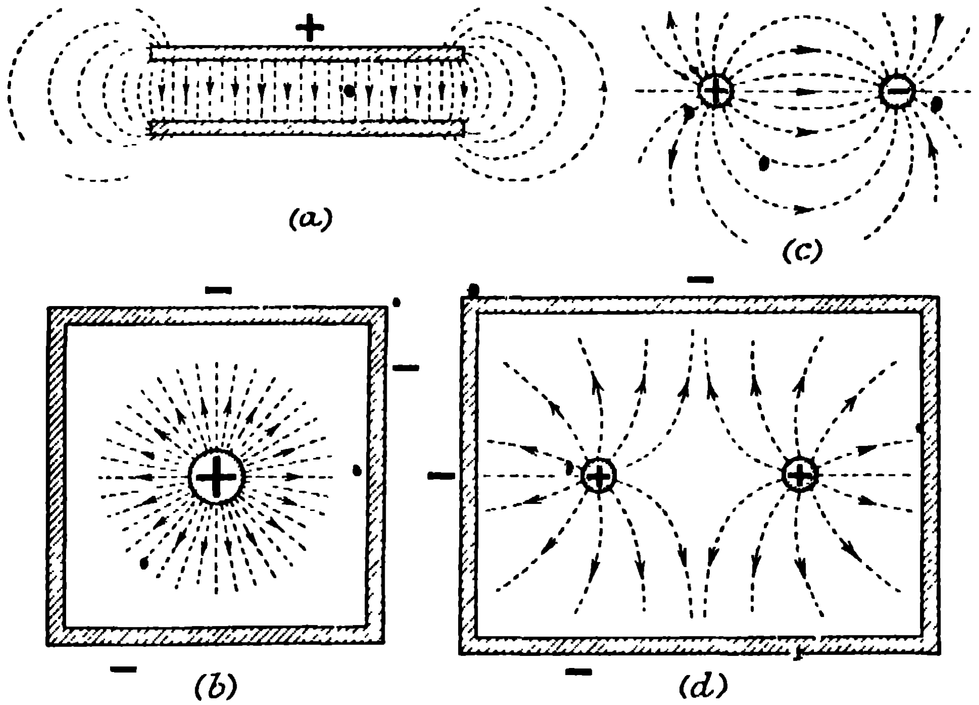


FIG. 46. ELECTRIC FIELDS

What happens in this two-dimensional experiment can safely be taken as indicating what happens throughout the space between the plates of a parallel plate condenser. The gypsum crystals show the direction of the electric flux.

In the second picture (Fig. 46 (*b*)) the terminals of the machine are connected to a tin-foil disc and a surrounding (distant) border; the disc can be taken as being a cross-section of an isolated charged sphere. The crystals arrange themselves in lines radiating from the centre of the disc.

In Fig. 46 (*c*) two discs are shown, together with the line pattern traced by the crystals under the same conditions. This gives an idea of the electric flux between two oppositely charged spheres or cylinders (such as telegraph conductors).

In Fig. 46 (*d*) the two discs have been connected to one terminal of the machine and the surrounding strip to the other.

If these lines traced out by the gypsum crystals are imagined to have an actual existence, being attached at one end always to a positive charge and at the other end always to a negative charge, and to be in tension along their length and in compression across their breadth, so that they are always trying to shorten and push outwards, the attractions and repulsions of electric charges will be explained, qualitatively, and if further the density of the lines crossing a normal area is made to depend on the magnitude of the forces produced, the lines may be used quantitatively as well. Thus an electric field or electric flux may be defined at any point by the direction of the lines and their number per square centimetre at that point.

Take a charge by itself of $+Q$ coulombs on an insulated brass ball, for example. All round this ball there is electric flux. The flux is symmetrical and is along the radii drawn outwards from the ball. This flux can be represented by lines drawn out along the radii from the centre through the space around. Each line represents so much flux, say, unit flux, outwards from a positive charge and inwards to a negative charge, like the lines of current flow from the electrode buried in the earth that was discussed in the last chapter. The direction of the flux is the direction in which a positive charge would move.

Let the total flux leaving the ball be $\Psi$ lines; this will be the total flux through any closed surface surrounding the ball. If a sphere is drawn of radius $r$ with its centre at the centre of the ball, the surface area of the sphere is $4\pi r^2$ and the flux density or lines per unit area through the sphere is $D = \Psi/4\pi r^2$ lines per cm.$^2$

If an electric charge $q$ is put at a point on the sphere it will experience a force along the radius, i.e. along the flux line, and if the charge on the ball is $Q$, the force on each charge will be proportional to the product of the two charges and inversely proportional to the square of the distance between them; i.e., the force is proportional to $Qq/r^2$. This law was established experimentally by a scientist named Coulomb. Actually if $Q$ and $q$ are in coulombs and $r$ is in centimetres the force is given in dynes by

$$\frac{Qq}{r^2} \times 9 \times 10^{18}$$

Let $q$ be imagined so small that its presence does not appreciably disturb the symmetry of the field. Then it is seen that both the flux density and the force on the small charge are inversely proportional to the square of the distance from the ball, and if a suitable unit is chosen for the flux the force can be measured by the product of the flux density and the charge $q$. This value is to be found by making $Dq$ equal to $(Qq/r^2) \times (9 \times 10^{18})$; i.e. it is found from the equation

$$\frac{\Psi}{4\pi r^2} q = \frac{Qq}{r^2} 9 \times 10^{18}$$

giving

$$\Psi = 4\pi Q \times 9 \times 10^{18}$$

The fact that the flux density can be used to measure the force on a charge at any point in a field, and the lines of flux used to find its direction is thus shown in the simple case of a charged ball. Actually it is found to hold in all cases however complicated.

Using the inverse square law, the electric field in the cases of Fig. 46 (*c*) and (*d*) may now be mapped out accurately by exactly the same construction as that used in Fig. 21, page 31, to find the lines of current flow.

The factor of $4\pi$ is kept separate from the rest because it has to do with the area of a sphere. But $9 \times 10^{18}$ is an enormous number, and it teaches an important fact about the forces between charges of electricity. The dyne is a very small force, about the magnitude of the force exerted by a small fly beating one of its wings. If a large enough number of small things be taken, they may amount to a great deal, in this case nearly ten thousand million tons. This would be the force exerted between two charges of 1 coulomb each if they could be brought to a distance of 1 cm. from each other. Clearly that is an impossibility. The force of repulsion or attraction would be very great with charges much less than one coulomb, so that a charge greater than a tiny fraction of a coulomb cannot be accumulated on any conductor. A lightning flash discharging a cloud sometimes conveys as much as a coulomb. A charge of this size, collected in the cloud before the flash, makes the electric field so intense that it breaks down the insulation of a great thickness of air.

**9. Capacitance of a Parallel Plate Condenser.** An expression can readily be obtained for the capacitance of a parallel plate condenser. Let the area of the plates be $A$ sq. cm. and their distance apart be $d$ cm. If the charge on each plate is $Q$ coulombs, the total flux is $\Psi = 4\pi Q \times 9 \times 10^{18}$, and since this is uniformly distributed across the area of the plates and the lines go straight across from one plate to the other, the flux density $D$ at any point between the plates is

$$D = \Psi/A = 4\pi Q \times 9 \times 10^{18}/A$$

and this would be the force in dynes on a charge of one coulomb placed between the plates. To carry this charge a distance $d$ from one plate to the other would involve work to the extent of force × distance, i.e. $Dd$ in dyne-cm., or $Dd \times 10^{-7}$ in joules, and

$$Dd \times 10^{-7} = 4\pi Qd \times 9 \times 10^{11}/A$$

But the work done in carrying a charge of one coulomb against a p.d. of $V$ volts is $V$ joules, and if $V$ is the p.d. between the plates,

$$V = 4\pi Qd \times 9 \times 10^{11}/A$$

and the capacitance of the condenser, which is $C = Q/V$, is given in farads by

$$C = \frac{A}{4\pi d} \times \frac{1}{9 \times 10^{11}}$$

If the condenser consists of a number of plates $n$ as in Fig. 47, there will be $(n-1)$ electric fields involved and the capacitance as obtained by the above expression for one field must be multiplied by $(n-1)$.

If the dielectric constant of the insulating medium is $\kappa$ instead of unity for air, the expression must be multiplied by $\kappa$.

To get 1 farad would require an enormous area in sq. cm., and this is due to the factor $9 \times 10^{11}$ which comes from the measure of the force between two coulombs at 1 cm. distance.

It is not to be wondered at that a microfarad (= one-millionth of a farad) is a capacity of more usual occurrence.

If the dielectric is not air but some other insulator, the capacitance will be

$$\frac{\kappa A}{4\pi d \times 9 \times 10^{11}} \text{ farads}$$

The flux per cm.$^2$ is $\frac{Q}{A} \times 4\pi \times 9 \times 10^{18}$ as before and as

$$\frac{Q}{V} = \frac{\kappa A}{4\pi d \times 9 \times 10^{11}}$$

the flux per cm.$^2$ is $\frac{Q}{A} \times 4\pi \times 9 \times 10^{18}$

$$= \kappa \frac{V \times 10^7}{d} = D$$

But $(V/d) \times 10^7$ is the force in dynes on 1 coulomb, so the force on 1 coulomb is no longer the number of lines of flux per cm.$^2$ between the plates.

The force on 1 coulomb is then $D/\kappa$ or

$$\text{field strength} = \varepsilon = D/\kappa$$

$$= \frac{\text{Flux per cm.}^2}{\text{Dielectric constant}}$$

In other words, flux density $D = \kappa$ times field strength.

10. **Electrostatic Voltmeters.** It is seen that the interesting parts of the phenomena are what occur in the field. For given charges the forces depend on the material occupying the field; the capacitance also depends on this material. The energy stored in the condenser is

$$\tfrac{1}{2}QV = \tfrac{1}{2}\frac{Q^2}{C}$$

as has been seen.

Let it be supposed that a condenser is charged in the following way. Start with the plates in contact, and from rubbing them together or otherwise, one of them gets a charge $+ Q$ and the other a charge $- Q$. As they are in contact these two charges neutralize one another's fields, and there is no energy in the arrangement as the capacitance $C$ is infinitely great with $d = 0$; $Q^2/C = 0$. Now draw them apart, and the condenser is charged as before. The net work done must be the same whichever way the condenser is charged. The work is done in this second way of charging the condenser by the mechanical force required to draw the charges apart against their mutual attraction.

The total work done is $\frac{1}{2}(Q^2/C)$ joules, and in any part of the pulling apart, the work done is $\frac{1}{2}Q^2 \times$ change in $1/C$, for that is the increase in the energy stored. As the plates are pulled apart, $C$ diminishes and so $1/C$ increases proportionately to the distance apart of the plates. The force pulling the plates together is proportional to $Q^2$ and therefore to $V^2$. This is the principle of a voltmeter known as the *attracted disc electrometer*. This is the earliest pattern of electrostatic voltmeter and is the simplest in action, but it is not sensitive to small potential differences, and this type is now only used for high potentials. In it there are two parallel plates which are connected to the conductors whose potential difference is to be measured. The central part of one plate is movable, and the force required to hold it in position, against the pull on it, is measured.

It is quite simple to find an expression for the actual force of attraction in the parallel plate case. If this force is $F$ dynes and there is a charge $Q$ coulombs on each plate, then if the plates are allowed to move towards each other the flux density and therefore the force remain unchanged, as the plates move, and the work done as the plates come together is $Fd$ ergs. This must be equal to the energy stored in the condenser, i.e. $\frac{1}{2}CV^2 \times 10^7$ ergs. Thus

$$Fd = \frac{1}{2}CV^2 \times 10^7$$

and using the expression for $C$ found above,

$$Fd = \frac{1}{2}\frac{A}{4\pi d} \times \frac{1}{9 \times 10^{11}} \times V^2 \times 10^7$$

$$F = \frac{AV^2}{8\pi d^2} \times \frac{1}{9 \times 10^4} \text{ dynes}$$

The energy in the condenser is stored in the electric field, and it is of interest to find its value in terms of the flux density $D$.

The energy is given in ergs by

$$Fd = \frac{2\pi d Q^2}{A} \times 9 \times 10^{18}$$

Writing

$$Q = \frac{DA}{4\pi \times 9 \times 10^{18}}$$

this gives

$$Fd = \frac{D^2}{8\pi} \times Ad \times \frac{1}{9 \times 10^{18}}$$

Now $Ad$ is the volume of the dielectric (air in the case considered) and the result can be written

$$\left.\begin{array}{l}\text{Energy stored per unit}\\ \text{volume of the electric}\\ \text{field}\end{array}\right\} = \frac{D^2}{8\pi} \times \frac{1}{9 \times 10^{18}}\ \text{ergs}$$

If the dielectric constant is $\kappa$, the energy stored is $\kappa$ times as great.

In the attracted disc electrometer or voltmeter the pull on the plates would draw them together. Such a movement would increase the capacitance of the condenser and therefore diminish the energy stored in the field for a given charge. If the plates move towards one another the diminution of stored energy provides the energy for the work spent in drawing them together.

The same principle in a different way is used in the *quadrant electrometer*, which is used practically in what is called the *multicellular* form. Several vanes are mounted parallel to one another on a spindle which is suspended by a fine wire with the spindle vertical, Fig. 47. The vanes swing in and out of the gaps between a series of horizontal metal plates shaped like a pair of opposite quadrants of a circle. The vanes and the quadrants form in this way the plates of a condenser whose capacitance varies with the position of the vanes in relation to the quadrants. The energy of the condenser thus formed is $\frac{1}{2}Q^2/C$, and as before the force of attraction between the plates of the condenser tends to increase the capacitance. This time it is by increasing the area of the plates of the condenser, by drawing the vanes into the quadrants. The pull is proportional to the square of the charge or of the p.d. as before. The vanes are controlled by the torsion of the suspension, and they come to rest in such a position that the torque due to the wire balances the torque pulling the vanes into the quadrants. Several vanes are used to make the deflecting torque greater and thus obtain a sufficiently sensitive instrument.

A diagram of the lines of flux between the plates and near the edge of one of the vanes on a quadrant electrometer is shown in Fig. 47A. The lines in the part *A* are those which appear to pull the plates together. The lines in part *B* are the lines which appear to pull the movable vane further in.

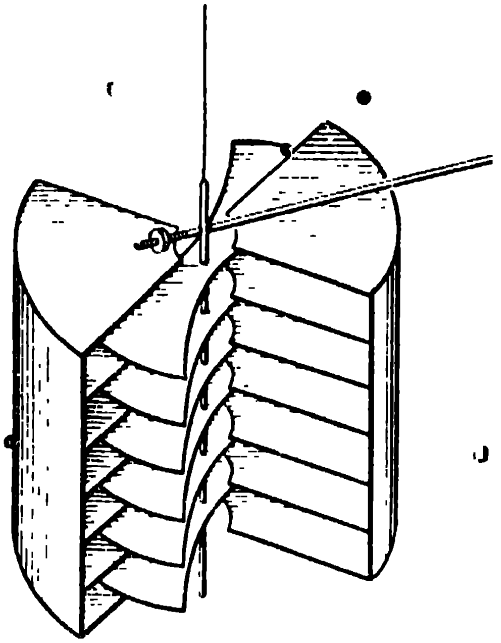


FIG. 47. ELECTROSTATIC VOLTMETER

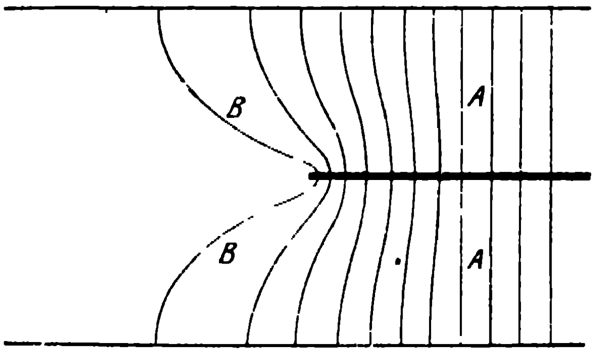


FIG. 47A. ELECTRIC FIELD BETWEEN MOVING VANE AND FIXED PLATES

Electrostatic voltmeters have two merits. The first is that they do not draw a current from whatever d.c. potential difference they are connected to. A very small charge flows into them when they are first connected, and then the current ceases. The potential difference is measured without making the slightest change in any current which may be flowing in any part of the circuit. The other merit is that the pull which gives the reading is the same if the connections are reversed, so that they can be used for alternating potential differences.

In this case they take only the very small current which flows through the capacitance of the voltmeter.

11. **Insulation Resistance of Condenser.** When a condenser has been charged and left insulated, the charge gradually leaks away because of the trace of conductivity which is possessed by even the best insulators. The quality of the dielectric can be tested by observing the rate of fall of the p.d. The current through the insulation is $I = V/R$. $R$ = insulation resistance. $V$ = p.d. Also current $I$ = rate of fall of charge $Q = C \times$ rate of fall of p.d. $= C\dot{V}$, so $C\dot{V} = V/R$.

The rate of fall of p.d. = fall of p.d. per sec. $= \dot{V} = V/CR$.

If the condenser is connected to an electrostatic voltmeter, and charged and then insulated, and the reading of the voltmeter taken every minute, the value of $R$ can be found. Plot a curve showing $\dot{V}$ = drop of volts per min. ÷ 60 against mean p.d. during that minute. A straight line graph will be obtained. The slope of the line gives the value of $CR$ and hence $R$, knowing $C$.

If $\rho$ is the resistivity of the dielectric in ohm-cm., the insulation resistance of the dielectric is $R = \rho(d/A)$ ohms, $d$ = thickness of dielectric in cm. and $A$ = area of plates in cm.$^2$ Also, we know the capacitance of the condenser in microfarads is

$$C = \frac{A\kappa}{4\pi d \times 9 \times 10^5}$$

Consequently we have

$$CR = \rho\frac{d}{A} \times \frac{A\kappa}{4\pi d \times 9 \times 10^5}$$
$$= \frac{\rho\kappa}{3{\cdot}6\pi \times 10^6}\ \text{ohm}\,\mu\text{F}.$$

This is the same for all condensers using the same dielectric whatever their capacity. The product $CR$ for a condenser is then a direct test of the quality of the insulation. The Post Office specification for paraffin paper condensers requires $CR = 300$ megohm-$\mu$F. measured after the application of 300 V. for one minute.

The insulation resistance may also be measured by the direct deflection of a sensitive moving coil instrument which has been calibrated.

## CLASS EXPERIMENTS
### ON THE SUBJECT OF CHAPTER II

1. A Condenser, charged to a measured p.d., is discharged through a ballistic galvanometer. The throw varies as $V$ and is hence proportional to $Q$ (Fig. A).

Repeat with various values of $C$ and $V$ and plot curves showing the variation of the throw with both $C$ and $V$.

Insert resistances in series with the condenser. The value of the throw is not altered. The resistance has reduced the current but not, of course, the quantity. This experiment proves that the ballistic galvanometer measures quantity of electricity provided that the quantity passes as a current of short duration.

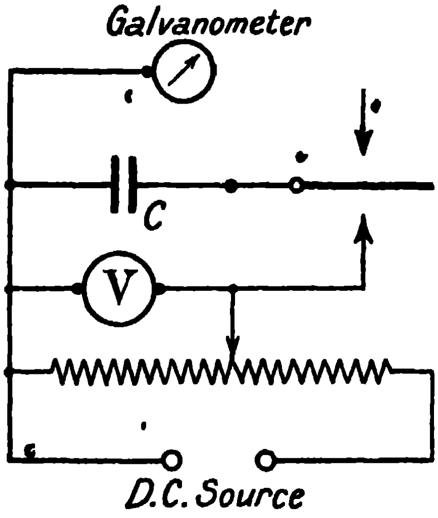


FIG. A

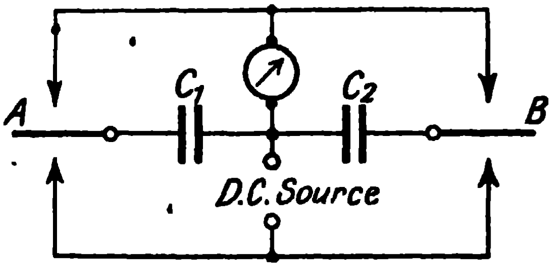


FIG. B

2. Comparison of Capacitances by Throw of Galvanometer.

Charge the condenser $C_1$ by depressing switch $A$ (Fig. B). Discharge $C_1$ through the galvanometer by raising switch $A$ and note the throw. Repeat with condenser $C_2$ by operating switch $B$. The capacitances of $C_1$ and $C_2$ should be proportional to the throws obtained.

3. Test of the Insulation of a Condenser by the Ballistic Galvanometer.

Using the circuit arrangement of Experiment 1, throw is measured immediately on charging and after insulation for measured times.

The fall of charge per sec. = Discharge current = $V/R$. Hence $R$ is found.

Shunt condenser with very high resistances (a few megohms) and repeat. Use results to check values of shunting resistances.

4. Measurement of Capacity, by method of Fig. 32.

5. Comparison of Condensers by Bridge Method (Fig. C).

If charged by currents proportional to capacitances the p.d.'s rise at same rate.

$$\frac{C_1}{V/R_1} = \frac{C_2}{V/R_2} \text{ i.e. } C_1R_1 = C_2R_2$$

Repeat with a.c. instead of d.c. source, omitting switch from circuits and replacing the d.c. by an a.c. detector (telephones or vibration galvanometer).

6. Current through Condenser from a.c. Source.

A hot-wire (or other a.c.) ammeter is connected in series with a condenser of capacitance $C$ across an a.c. source of frequency $f$ (Fig. D). The p.d. across the condenser is measured by an electrostatic voltmeter. Check that $I = 2\pi fCV$ for various values of $CV$ and, if possible, $f$.

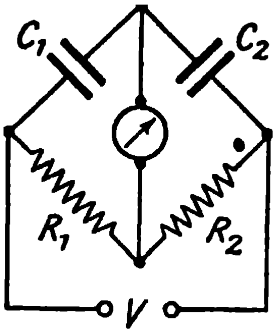


FIG. C

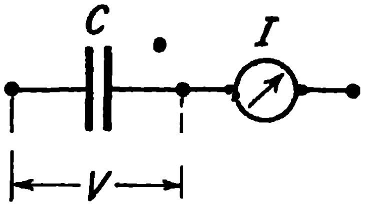


FIG. D

7. Current through Resistance and Capacitance in Series.

Repeat the last experiment with a resistance in series with $C$, measuring the three p.d.'s, $V_1$ across $C$, $V_2$ across $R$, and $V_3$ across $C$ and $R$.

Check that $I = \dfrac{V_3}{\sqrt{(R^2 + 1/\omega^2C^2)}}$ and draw the vector diagram, that is, draw a triangle of which the sides are proportional to $V_1$, $V_2$ and $V_3$. The angle between the first two sides should be a right-angle.

8. Make a similar experiment with resistance and capacitance in parallel. Measure $I$, the total current, and $V$, the p.d., across the resistance and the condenser.

Check that $I = V\sqrt{(1/R^2 + \omega^2C^2)}$.

## EXERCISES

1. The current-time curve of a condenser during charge is as follows—

| $t$ | 0 | 0·005 | 0·01 | 0·015 | 0·02 | 0·025 | 0·03 | |
|---|---|---|---|---|---|---|---|---|
| $i$ | 100 | 61 | 37 | 22 | 14 | 8 | 5 | mA. |

Draw the charge-time curve.

2. A p.d. of 2 000 V. is established across the terminals of a condenser of 10 $\mu$F. What charge does the condenser take? (0·02 C.)

3. A parallel plate air condenser has a capacitance of 50 $\mu\mu$F. What will its capacitance be if the space between the plates is completely filled with sulphur? (200 $\mu\mu$F.)

4. Two condensers of capacitances 0·5 and 0·4 $\mu$F. respectively are connected (*a*) in series, (*b*) in parallel. What is the capacitance of the combination in each case? ( (*a*) 0·222 $\mu$F.; (*b*) 0·9 $\mu$F.)

5. A condenser of 2 $\mu$F. is charged to a p.d. of 50 V. Find the energy stored. (0·025 J.)

6. A charged condenser has a stored energy of 0·02 J. If its capacitance is 4 $\mu$F., find its p.d. and its charge. (100 V.; 400 $\mu$C.)

7. Draw a current-time curve of the type shown in Fig. 39, making the maximum value of the current 20 mA. and the frequency 50 cycles per sec.

Supposing that this is the current in a condenser of capacitance of 2 $\mu$F., draw curves showing the variations with time of the charge in the condenser and the p.d. across it.

(Max. values: $Q = 63{\cdot}6\ \mu$C. $V = 31{\cdot}8$ V.)

8. A p.d. of 220 V. r.m.s. is established across a condenser of 0·05 $\mu$F. Find the current into the condenser when the frequency of the p.d. is (i) 50 cycles per sec., (ii) 500 cycles per sec., (iii) 5 000 cycles per sec. ( (i) 3·46 mA.; (ii) 34·6 mA.; (iii) 346 mA. r.m.s. in each case.)

9. Draw the vector diagram in the cases of Question 7 and Question 8.

10. The reactance of a condenser at a frequency of 50 cycles per sec. is 100 Ω. What is the capacitance of the condenser? (31·8 $\mu$F.)

11. A resistance of 1 000 Ω. and a condenser of 4 $\mu$F. are connected in series across a supply of 200 V., 50 cycles per sec.

Find the current which flows and the p.d. across the resistance and across the condenser.

Draw all these quantities as a vector diagram.

(156 mA.; 156 V.; 124 V.)

12. If, in Question 11, 200 V. is the r.m.s. value of the supply p.d., find (i) the power taken from the supply; (ii) the maximum value of the energy stored in the condenser. (31·2 W.; 0·061 J.)

13. An a.c. source of 440 V. 50 cycles per sec. supplies 15 A. to a motor which takes 5·94 kW. What is the power factor of the motor? (0·9.)

14. Find the capacitance of a parallel plate air condenser having 11 plates, each of 50 cm.$^2$ area, the distance between plates being 2 mm. (221 $\mu\mu$F.)

15. A mica condenser consists of 17 rectangular copper foil plates 6·5 cm. × 5 cm. separated by mica of thickness 0·0075 cm. The measured capacitance is 0·04 $\mu$F. Find the dielectric constant of the mica. (6·55.)

16. A parallel plate air condenser of area 126. cm.$^2$ and distance apart of plates 0·5 cm. is charged to a p.d. of 500 V.

Find the flux density and the energy stored and the force of attraction between the plates. ($10^{10}$; 27·7 ergs; 55·5 dynes.)

## CHAPTER III

## ELECTROMAGNETICS—THE MOTOR EFFECT

1. **Magnetic Fields.** The attraction between two like charges of electricity has been studied in the last chapter. If two wires carrying currents in the same direction are suspended vertically close to each other a force of repulsion would be expected between them as they are charged, and although the charges are moving this would not affect the repulsion arising from the electric field. This force is, however, extremely small, and actually the wires are found to *attract* each other with a very much greater force. The longer the wires and the greater the current the greater the total force.

If a compass needle is brought near a vertical current-carrying conductor, it is deflected to a direction tangential to the horizontal circle drawn through the needle with the wire as centre. The greater the current and the nearer the needle to the wire the greater the force deflecting the needle.

These effects are due to the magnetic field of the current, which must not on any account be confused with the electric field of the moving electricity constituting the current. Always there are both electric fields and magnetic fields when electricity is in motion. Either may be of much greater importance than the other, but sometimes they are of equal importance. In the charging and discharging of a condenser the electric field is of far greater importance than the magnetic field, which however is still present round the wires leading to the condenser, and even in the dielectric of the condenser itself. In the cases to be considered in this and the next chapter the magnetic field is of far greater importance than the electric field. But in the transmission of electric energy from town to town over long transmission lines and in the propagation of wireless waves round the Earth, both fields come in and it cannot be said that one is of greater importance than the other.

Electricity in motion, then, has an influence on the space around it, in addition to producing an electric field. This influence is directed along lines linked with the current and is called the *magnetic field* of the current. The magnetic field may be mapped out by iron filings. Take a vertical wire threaded through a piece of card which is held horizontally. Scatter iron or steel filings on the card and pass a current through the wire. The filings will arrange themselves in lines looped round the wire (Fig. 48), particularly if the card is slightly tapped to help the filings to arrange themselves. The lines are like the streamlines of an eddying fluid rotating round the wire. A small iron or steel needle, pivoted at its centre and brought near the wire, will set itself along the direction of these lines, and this provides a means of finding the direction of these lines more accurately and at a greater distance from the wire than is possible with the filings. The effect is greater if the needle is "magnetized," i.e. if a compass needle is used. It can be shown that the filings have been magnetized and each sets itself as the compass needle would do in the same position.

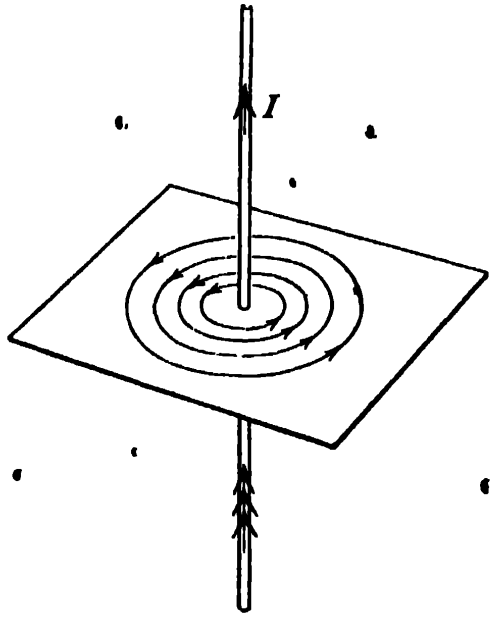


FIG. 48. MAGNETIC FIELD OF STRAIGHT CONDUCTOR

The pivoted iron wire or compass needle is pulled more vigorously into the line, the nearer it is to the wire carrying the current and the greater the current. This is shown by the liveliness and rapidity of its vibrations. If the wire is wound round to make a coil of several turns, the magnetic field is found to be quite strong for a moderate current, and it is easy to trace the direction of the field for a considerable distance around. The lines of the field will be found to be linked with the current as if there were a flow through the ring which returned back round the outside as is indicated in Fig. 49. Now a line has two directions, and it is worth while to examine whether there is anything to distinguish the two directions along a line in the field from each other. There is a forwards and backwards direction on these lines as can be seen from the behaviour of a compass needle. The end of the needle which points North will point along these lines in a particular way, and the end which points South will point the other way. It has been decided to call "the direction" of the lines the direction a north pointing end of a compass indicates, and the lines can be marked with arrowheads to show this direction. The direction of the lines and the direction of the current producing them are connected by a simple rule. The direction of the lines round the current is the direction a corkscrew (or any other right-handed screw) must be turned in order that it should travel in the direction of the current. Another way of stating the same thing is this: grasp the conductor with the fingers of the right hand, placing it so that the thumb points along the direction of the current; then the fingers point round the wire in the direction of the lines in the magnetic field.

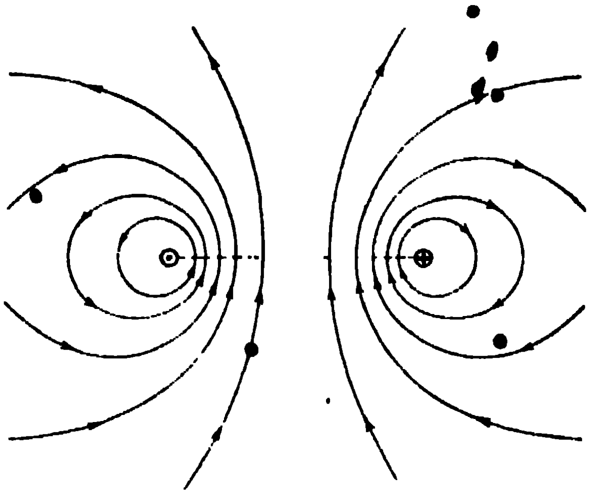


FIG. 49. MAGNETIC FIELD OF CIRCULAR COIL

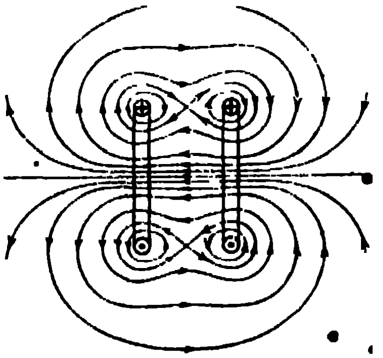


FIG. 50. COILS WITH CURRENTS IN SAME DIRECTION

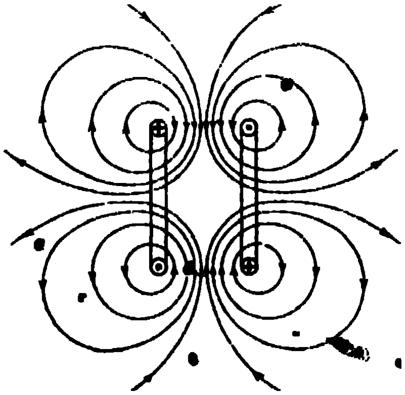


FIG. 50a. COILS WITH CURRENTS IN OPPOSITE DIRECTIONS

If two similar ring coils are placed parallel to one another on the same axis as in Figs. 50 and 50*a*, the lines mapped out by a compass or filings will be different, according to whether the current flows the same way round the two coils or in opposite ways. When the direction of the current is the same in both, the lines of one are mostly carried on through the other (Fig. 50), and when the two directions of the currents are opposed the two sets of lines turn aside and pass out between the rings (Fig 50*a*). If wire is wound in a uniform layer on a tube and a current is passed through the wire, the successive turns behave like the pair of coils just mentioned, and the lines of the field are carried on from one turn to the next

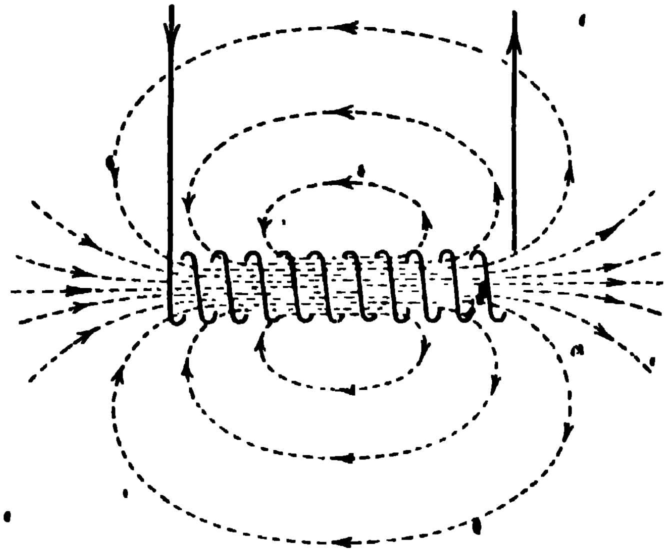


FIG. 51. MAGNETIC FIELD OF SOLENOID

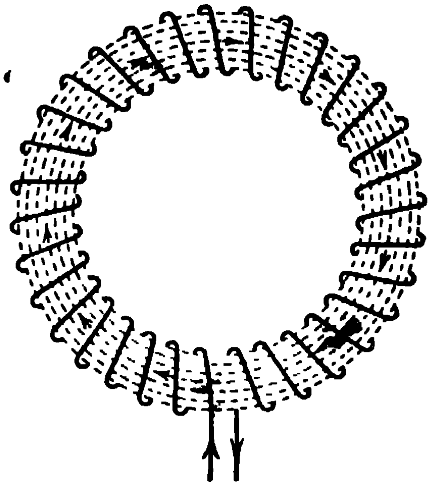


FIG. 52. MAGNETIC FIELD OF TOROID

and, diverging near the end of the tube, return and form complete loops as in Fig. 51.

Such a tube shaped coil is called a *solenoid.* Fairly intense magnetic effects can be obtained with such a coil. An iron rod hung with its lower end just inside a vertical solenoid is pulled down, and iron filings show the directions of the lines to a good distance from the windings.

But the simplest field of all is produced by a *toroid,* as a solenoid turned round so that its ends meet is called. In this case, shown in Fig. 52, the field is confined to the space within the windings, and if the diameter of the toroid is large compared with the diameter of the turns of wire, as is usually the case, the field is nearly uniform over the cross-section.

These effects are much increased if an iron core is put in the winding. The lines in the field tend to crowd into the iron and to be led through it rather than through the space round the iron. This can be demonstrated most clearly in the case of the solenoid. If the iron core projects out of the windings and is bent round so that its ends are near one another to form an air-gap, very intense magnetic fields may be produced in the air-gap. The pull on a piece of iron may be made very great by this means. If the ends only have a small gap between them, the majority of the lines linked with the coil through the iron pass across the gap. The magnetic fields are also increased if the current is increased or if the number of turns of wire is increased.

The magnetic field of a bar magnet or of a compass needle may be mapped in the same way (placing the card of iron filings over the magnet) and will be found to be very similar to that of the solenoid, although no arrangement of the filings in line will be found along the length of the magnet.

The lines mapped out by the iron filings give the direction of the magnetic forces produced by the movement of the electricity in the wires. They may also be used quantitatively to measure the forces, in the same way as the lines in the electric field were used. The magnetic field may therefore be determined by the direction and number per cm.$^2$ of these lines of magnetic flux.

2. **The Magnetic Circuit (Air-core).** In order to investigate the circumstances of the production of magnetic fields quantitatively, let an arrangement be set up as indicated in Fig. 53. A toroid is supplied with current through an ammeter and a variable resistance, and included in the circuit is a switch *S*. Inserted into the toroid, through an opening in the winding, is a search coil *C* which is connected to a sensitive ballistic galvanometer *G*. The coil *C* is arranged with its plane perpendicular to the axis of the toroid, so that the lines of flux thread through its turns of wire. It will be found that when the current is started in the toroid by closing the switch *S* the galvanometer will give a "throw." When the current is stopped by opening the switch the same throw will be obtained, but in the opposite direction. This phenomenon, which is of the greatest importance, will be investigated in the next chapter; here it is used only to indicate when the field strength in the toroid, i.e. the number of lines per cm.$^2$ through the search coil, is the same.

Various toroids are used of different axial lengths (i.e. mean lengths of flux lines), different shapes and areas of cross-section ($a$), and different numbers of turns ($N$) in different numbers of layers, but the same search coil is used in each, and in each the cross-sectional diameter is small compared with the diameter of the toroid. For each toroid the current $I$ as read by the ammeter $A$ is adjusted by the variable resistance $R$, until the same throw is obtained in the galvanometer $G$ when the switch $S$ is closed. Under these circumstances it is clear that the same magnetic field has been produced in the toroid in each case.

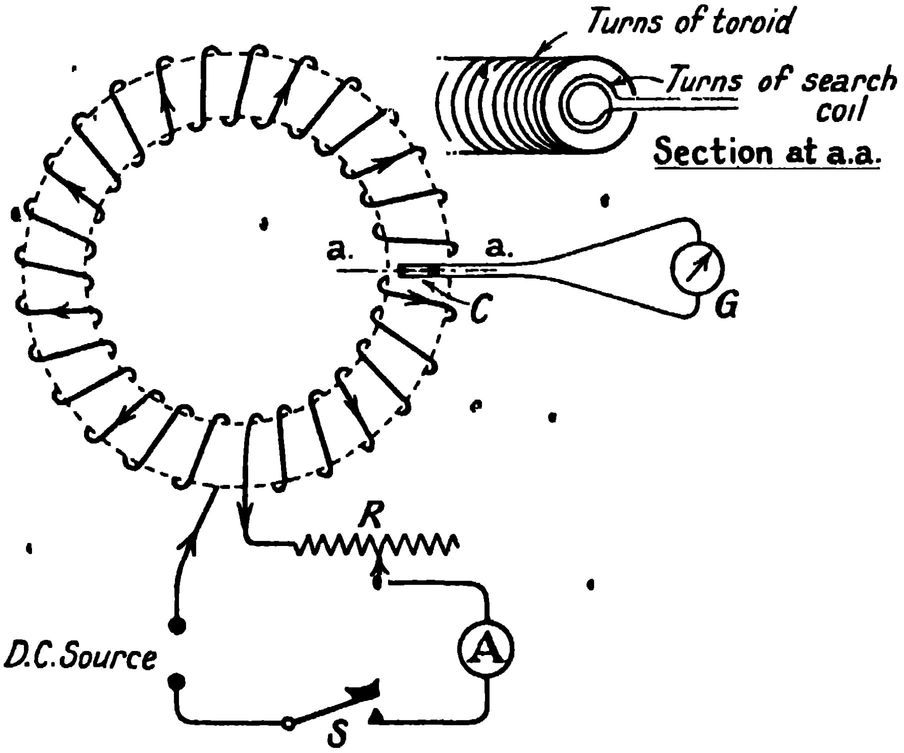


FIG. 53. EXPERIMENTS ON MAGNETIC FIELD STRENGTH

It will be found that in all the cases the product $IN/l$ is a constant. The cross-sectional area $a$ has no effect. It follows that the field is directly proportional to the number of turns in the toroid and to the current, i.e. directly proportional to the *ampere-turns*, and inversely proportional to the length of the lines of flux.

In order to make the system of units fit all requirements, the magnetic field or flux density $H$ in *lines per cm.*$^2$ or *gauss* is actually measured as $4\pi/10$ times the ampere-turns per cm. length of the flux lines, i.e.

$$H \quad \frac{4\pi}{10} \cdot \frac{IN}{l}$$

The total lines of flux $\Phi$, since the flux is uniform across the cross-section of the toroid, is $\Phi = Ha$, i.e.

$$\Phi = \frac{4\pi}{10} \cdot \frac{IN}{l} \cdot a$$

This can be written in the form

$$\frac{4\pi}{10} \cdot IN = \Phi \cdot \frac{l}{a}$$

$(4\pi/10).IN$ is known as the *magneto-motive-force* (an unfortunate term perhaps, as it is not in any way the sort of force that is understood in mechanics). It is the cause of the flux.

$\Phi$ is the total flux and $l/a$ is termed the *reluctance* of the toroid.

The expression becomes then—

Magneto-motive force = flux × reluctance

As has been seen above, magnetic flux always proceeds in closed circuits. In this way it resembles an electric current, and a very useful comparison may be made of the current flow in an electric circuit and the flux in a magnetic circuit. In the former case, by Ohm's law,

| | |
|---|---|
| Potential difference<br>round the circuit<br>(electro-motive-force) | = current × resistance |

Compared with the law for the flux in the toroid, it is seen that electro-motive-force corresponds with magneto-motive-force, current with flux, and resistance with reluctance.

Although established here for the case of the toroid, this magnetic circuit idea can be used in all cases, however simple or however complicated, but sometimes the mathematical difficulties in its application are great.

For instance, consider the straight wire of Fig. 48. It is required to find the field at a distance $r$ cm. from the wire. Here there is only one "turn" (of infinite radius) and the magneto-motive-force (m.m.f.) is $4\pi I/10$. Imagine a tube of uniform cross-section $a$ cm.$^2$ and mean radius $r$ cm. surrounding the wire. The reluctance of the tube is $l/a = 2\pi r/a$. The flux in the tube is accordingly $\Phi = (4\pi I/10) \div (2\pi r/a) = 2Ia/10r$. The field strength or flux density is $H = \Phi/a$ and hence

$$H = \frac{2I}{10r} \qquad \text{lines per cm.}^2$$

The case of the solenoid (Fig. 51) may be compared with that of earth conductors (Fig. 20), provided that the length $l$ of the solenoid is much greater than its diameter. In the same way as the resistance of the earth was negligible, the reluctance of the total space outside the solenoid is negligible, and all that need be taken into account is the reluctance of that part of the circuit which is inside the solenoid, which is $l/a$. The m.m.f. is $4\pi IN/10$; and the flux therefore is

$$\Phi = \frac{4\pi}{10} \cdot NI \cdot \frac{a}{l}$$

For points inside the solenoid where the cross-sectional area of the total flux is $a$ and the flux density is uniform, $H$ is $\Phi/a$, i.e.

$$H = \frac{4\pi}{10} \cdot \frac{NI}{l}$$

as in the case of the toroid. The solenoid can in fact be looked upon as part of a toroid of infinite diameter.

**3. Forces on Currents in Magnetic Fields.** (i) FORCE ON STRAIGHT CONDUCTORS. In general a conductor carrying a current in a magnetic field experiences a mechanical force. The force between two parallel conductors noted at the beginning of this chapter arises from the fact that the current in each conductor is situated in the magnetic field due to the other.

The law governing these forces can be investigated with the help of the toroids used in the last section. If a flimsy wire is introduced into the gap in the turns instead of the search coil and, the switch $S$ being permanently closed in all these experiments, a current is passed through the wire in one direction, the wire will tend to be forced radially out of the toroid. If the current is reversed, the wire will be forced radially further into the toroid.

Measurements can be made if the flimsy wire is replaced by a coil side as indicated in Fig. 54. The coil side *AB* should be long enough to clear the toroid of largest cross-section, and the coil should be long enough to move freely when assembled with the toroid of largest diameter. The coil is suspended between a spring *S* and a torsion lead *T* by two fine steel wires or phosphor bronze strips *ww*. The cylinder *W* can be moved along the arm to act as a counterbalancing weight. The torsion head is used to bring the coil side back to the centre of the cross-section of the toroid. The torque in the wires *w* is proportional to the movement of the torsion head; and the torque required to bring the coil side back is equal to the torque due to the force on the coil side. Hence since this is the only force acting, the movement of the torsion head is proportional to the force on the coil side when it carries a current in the magnetic field of the toroid. The current is led to the coil by means of the wires *w*, and the length *l* cm. of that part of the coil side which is in the field *H* of the toroid is noted. (It will be different for different toroids.) The field *H* is estimated by a knowledge of the current in the toroid and its turns per centimetre length, and the current *I* amperes through the coil side is measured. In this way it is established

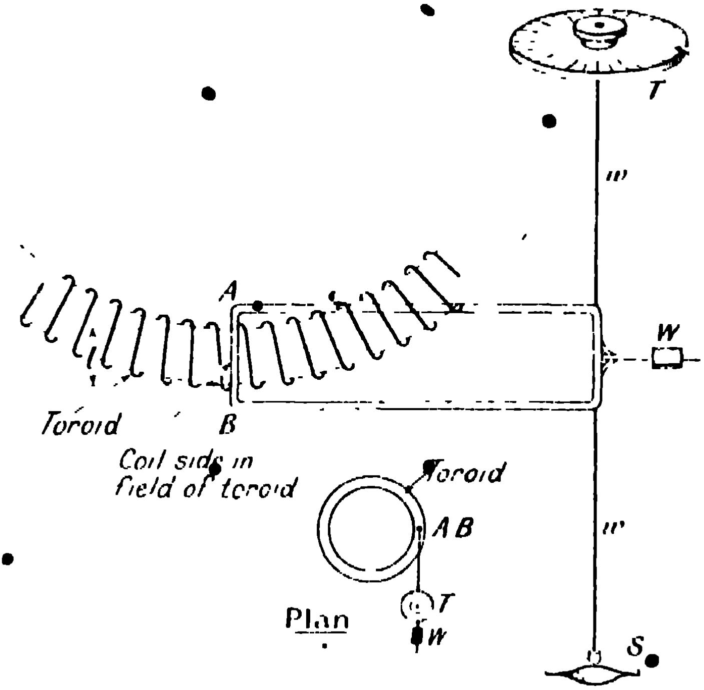


FIG. 54. EXPERIMENTS ON FORCE ON CURRENT IN MAGNETIC FIELD

experimentally that the force is proportional to the product of the flux density, the current in the coil side and the length in the field. This is *Ampere's Law.* (Actually the units are such that

$$F = \frac{HIl}{10} \text{ dynes}$$

if the coil has only one turn. $I$ is in amperes and $l$ in centimetres. If there are $n$ turns the force is $n$ times as great, as the total length of conductor in the field is now $nl$ cm.

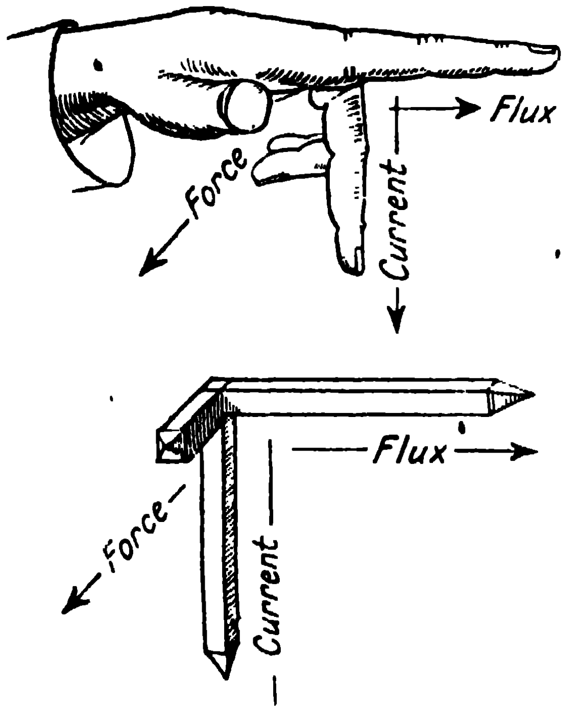


Fig. 55. Left-Hand Rule

The direction of the force is perpendicular both to the flux and to the length of the conductor. It can be found by what is known as the *left-hand rule*, Fig. 55. The thumb and first two fingers of the left hand are set at right-angles to each other, and the hand is turned so that the first finger points in the direction of the field and the second finger points in the direction of the current. The thumb will then point in the direction of the force (motion). This rule should be used to check the direction of the force in the above experiment.

If the lines of flux are mapped round a straight wire carrying a current across a magnetic field some explanation of this force may be seen.

The flux is obtained by combining the flux due to the current with that due to the independent field. These are shown separately in Fig. 56. In Fig. 56A they are combined. It is seen that where the two fluxes are in the same direction the resultant lines are crowded, and where they are opposed the lines are wider apart. The force seems to be due to the lines trying to shorten and to push one another aside.

(ii) Torque on Coils. If a coil of wire be placed in a region of magnetic flux, that is, in a magnetic field, there will be a torque or twist on the coil which tries to turn the coil so that the flux due to the coil is in the same direction through it as the flux in the field. This twisting couple is easily found from the law of force of Ampere. In Fig. 57 are shown the elevation and plan of a rectangular coil set up vertically in a uniform field of magnetic flux. This flux is directed horizontally from left to right and has strength $H$ lines per cm.$^2$ A current $I$ amperes flows in the $n$ turns of the coil, and looked at from the point of view taken in the drawing the flux through the coil due to the current in it is in the direction of the arrow shown in the plan. In the vertical sides of the coil there are $n$ conductors, each of length $h$ cm. and each carrying a current $I$ amperes at right-angles to the flux $H$ lines per cm.$^2$ There is a force on each coil side therefore of strength

$$F = \frac{HnIh}{10} \quad \text{dynes}$$

and the directions of the forces are in the directions shown by the left-hand rule. These two equal forces form a couple of torque $= Fe$, if $e$ cm. is the distance between the lines of action

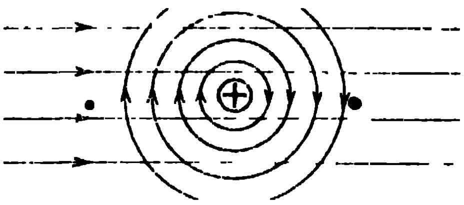


FIG. 56. UNIFORM FIELD AND FIELD DUE TO STRAIGHT CONDUCTOR

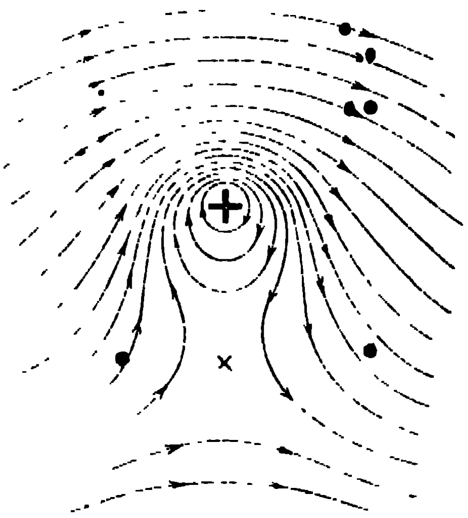


FIG. 56A. COMBINED FIELDS

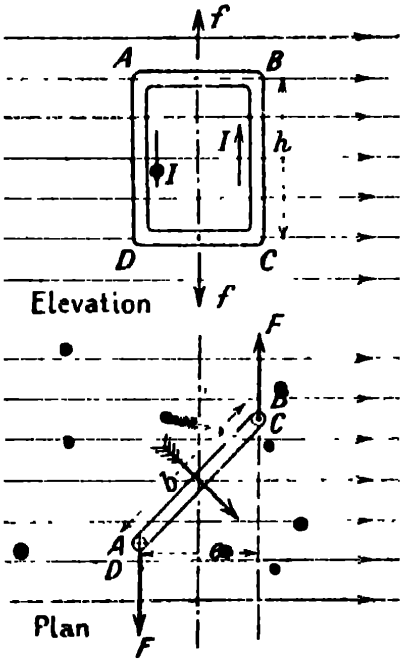


FIG. 57. TORQUE ON COIL

of the forces. Clearly the direction in which these forces try to turn the coil is such as to bring the two fluxes into the same direction. The greatest torque is when $e$ is greatest, that is when $l = b$, the breadth of the coil, or when the plane of the coil is along the direction of the field, and the torque is then

$$G = \frac{Hnlh}{10} \cdot b \text{ dyne-cm.}$$

But $hb$ = the area of the coil, and so the maximum torque is $\frac{HI}{10} \times$ total area of all the turns.

It should be noted that the forces $f$ acting on the sides $AB$ and $CD$ do not give any torque in this case because they are equal and opposite and act in the same straight line. The same result would be obtained if a coil of any other shape than a rectangle were taken. The torque would be found to have a maximum value

$$G = \frac{HIA}{10} \text{ dyne-cm.}$$

in such a direction as to turn the flux due to the coil into line with the flux of the field.

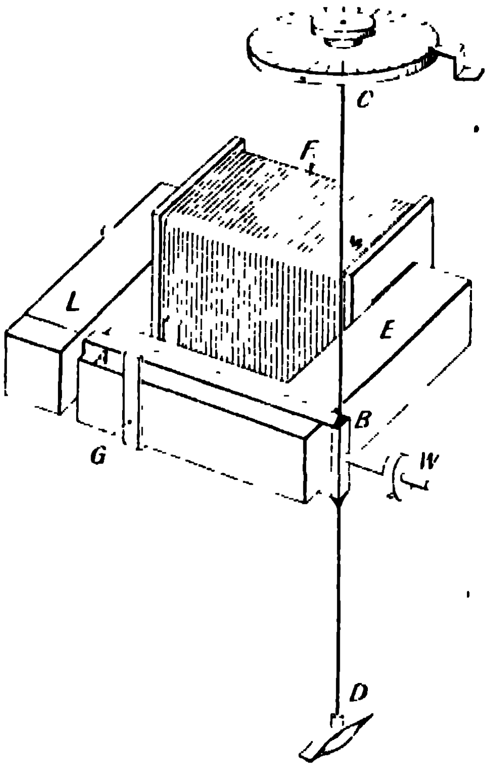


FIG. 58. APPARATUS FOR EXPERIMENTS ON MAGNETIC FIELDS

(iii) AN APPARATUS FOR THE INVESTIGATION OF LAWS OF FORCES AND FIELDS. The production of magnetic fields in air-gaps and the force acting on a conductor carrying a current in a magnetic field can also be conveniently studied by the more robust apparatus shown in Fig. 58.

A rectangular coil $AB$ 10 × 2 cm. of 10 turns 30 S.W.G. d.s.c. copper wire is suspended by two lengths of 30 S.W.G. steel wire stretched between the torsion head $C$ and the spring $D$. The coil is balanced by the weight $W$. Current is led to and from the coil by the two lengths of steel wire. The end $A$ of the coil moves freely in the air-gap in $LE$ a rectangle of soft iron bar about 2 cm. × 2 cm. section and about 10 cm. side. On one side of the iron rectangle is a coil *F* wound in sections of 50, 100, 150, 200, and 500 turns of 22 S.W.G. d.c.c. copper wire. The supporting framework is omitted for the sake of clearness. *G* is a strip of metal bent round a limb of the iron bar to act as stops limiting the swing of the coil. A pointer fastened at *B* lies along the coil and is adjusted, by turning *C*, to lie along a mark on the iron core at *L*. The coil *AB* is wound on a thin copper frame to stiffen it. A current is passed through the coil *F* causing a magnetic flux across the gap at *L*. This current is read on an ammeter and kept constant. Current from another source is passed through the coil *AB* and the coil is deflected against one of the stops. It is brought back to the central position by turning the divided head *C*. This twists the wire *CB* and exerts a torque on the coil to balance the moment of the deflecting force due to the current passing across the flux at the gap. The twist of the head *C* is noted and plotted on a base showing the current in the coil *AB*. A straight line graph is obtained showing that the force on the coil side in the field is proportional to the current in the coil. If the magnetizing current in the coil *F* is increased the deflecting force is increased, showing that the force increases with increasing flux density in the gap. If the current in *AB* is kept constant and the current in *F* is varied and the reading on *C* is plotted on a base giving the current in *F*, a graph showing how the flux in the gap varies with the current producing it is obtained. The graph is not quite straight and the line for decreasing current does not lie along the line for increasing current, but above it, showing a sort of "backlash" in the production of the flux. This is due to the presence of the iron in the magnetic circuit.

The observations are now to be repeated, using a different number of turns, and it will be found that the same flux in the gap is obtained when the product of magnetizing amperes and turns is the same. The flux depends on the number of ampere-turns *AT* used in the coil *F*. The results of different measurements will agree and lie on the same curve if the measurements are carefully made and account is taken of whether the current is being increased or decreased. The flux depends on the ampere-turns and increases with it. Often it may be taken as proportional to the ampere-turns as a fair approximation. The behaviour of iron in a magnetic field will be examined later.

**4. Force Between Parallel Currents—The Ampere Balance.** The force between two parallel conductors carrying currents can now be estimated. If the distance apart of the conductors is $d$ cm. and the current in the first is $I$ amperes, the field due to this current at the second is, as has been seen above, $2I_1/10d$ lines per cm.$^2$ The force on a length $l$ cm. of the second conductor carrying a current $I_2$ amperes $(HIl/10)$ is accordingly $2I_1I_2l/100d$. Similarly the force on the first conductor due to the field of the second is also $2I_1I_2l/100d$, hence the force on either is

$$F = \frac{2I_1I_2l}{100d} \text{ dynes}$$

If $l$ is measured in feet and $d$ in inches and the force is required in pounds weight,

$$F = \frac{2I_1I_2l}{100d} \times \frac{12}{981} \times \frac{1}{454}$$

$$= \frac{0{\cdot}54I_1I_2l}{d} \times 10^{-6} \text{ lb. wt.}$$

If the currents are in the same direction the force is one of attraction; if in the opposite direction one of repulsion.

If the current in each conductor has the same value $I$ amperes, the force per foot run of the conductors is

$$F = \frac{0{\cdot}54I^2}{d} \times 10^{-6} \text{ lb. wt.}$$

where $d$ is in inches.

If the two wires are turned round to form coils the result is the same; the coils will attract each other if the currents are in the same direction in each, and repel each other if the currents are in opposite directions. This can in fact be gathered from the lines of flux of Fig. 50 and Fig. 50 ($a$).

The attraction and repulsion of coils carrying currents with their planes parallel is the principle used in Kelvin's Ampere Balance. A movable coil is hung on the arm of a balance so that it lies horizontally between two fixed horizontal coils. The same current $I$ flows in opposite directions in the upper and lower fixed coils. When the current $I$ also flows through the hanging coil it is attracted by one and repelled by the other of the fixed coils. This force is balanced by weights. If all the currents are reversed the pull is not reversed, and the formula for the balance weight in grammes is $W = AI^2$. $A$ is the instrument constant. $A$ can be found once for all by weighing the amount of copper or silver deposited in a given time by the current that balances, say, 1 g. weight, and then any other current can be found accurately by finding the weight to balance it. The balance will measure a.c. on the same scale as d.c.; it gives the root mean square value in the a.c. case.

5. **Magnetic Materials and Magnetism.** It was noted at the beginning of the chapter that if iron or steel is present in the magnetic circuit the magnetic effects are greatly increased. The same applies to the few other "magnetic" materials, cobalt, nickel, and some alloys.

If, for instance, the experiment of Fig. 53 is repeated with a toroid having an iron core, the search coil of course being wound on the iron and being made identical with that used before, the value of $IN/l$ for an equal throw of the galvanometer is found to be very much less than the value found previously for the air-cored toroid (perhaps only one-thousandth—depending upon the iron).

The iron has an enormous effect upon the flux produced by a given magneto-motive-force. Ampere showed that this would be explained if there were a circulation of electricity in the atoms of magnetic metals. Each atom would then behave as a small coil with a current in it and have lines of magnetic flux linked with it. In a magnetic field these atomic currents would tend to turn so that their fluxes came into line with the flux in the field. Thus there would be an increased flux as the ampere-turns in the atoms would add their m.m.f. to that of the ampere-turns producing the original flux in the field. The examination of the behaviour of the materials which show magnetic properties leads to several results of practical importance. The *magnetization*, as it is called, of iron can be studied with the apparatus shown in Fig. 58. Keeping the current in the moving coil constant at a convenient value, the torque on the suspension will be proportional to the flux round the iron circuit. If the ampere-turns in the coil $F$ are gradually increased from zero and the torsion head turned to bring the moving coil central at each stage, a measure of the flux through the iron for each value of the magneto-motive-force is obtained. A graph of flux on a base of m.m.f. would be like Fig. 59, curve $OA$. The flux increases at first proportionally to the m.m.f. but at a certain stage the ratio of flux to m.m.f. becomes much greater. Afterwards the increase of flux falls off and the flux reaches a nearly constant value at $A$. The relation between flux and m.m.f. is seen to be rather complicated. All magnetic materials show the same kind of behaviour to a greater or less degree. If the m.m.f. is now reduced gradually the flux is found to decrease to the value $OB$ when the current in the coil $F$ is zero. The iron core is left with a flux through it. It is "magnetized." This magnetization retained by the iron is called the *residual magnetization*. The amount of residual magnetization varies very much with the material used for the core. The percentage $\left(\frac{OB}{AG}\right) \times 100$ retained is called the percentage *retentivity* of the sample.

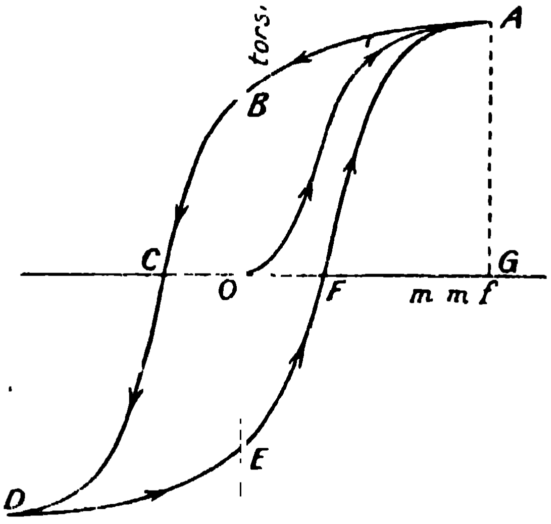


FIG. 59. HYSTERESIS LOOP

If a current is now passed through the magnetizing coil in the reverse direction and gradually increased the reverse m.m.f. removes the residual magnetization, and at a certain stage there is no flux in the core. The reverse m.m.f. $OC$ required to do this is a measure of the obstinacy with which the residual magnetization is held, and $OC$ may be used as a measure of what is called the *coercive force* of the material of the core. Increasing the reverse m.m.f. magnetizes the core in the reverse direction up to the value given by the point $D$. Decreasing the m.m.f. to zero and reversing again will remagnetize the core in the original direction. The flux follows the changes in m.m.f. along the line $DEFA$ exactly repeating the curve $ABCD$. The magnetization is seen to lag behind the changes in m.m.f. at every stage so that the complete curve forms a loop. This is called *magnetic hysteresis* and the loop the *hysteresis loop*. Hysteresis means "lagging." The important points about the magnetic behaviour of the core are seen to be—

(1) The value $AG$ of the maximum flux and the m.m.f. $OG$ required to reach this state.

(2) The residual magnetization $OB$.

(3) The coercive force $OC$.

The flux in the core is seen to tend to a maximum value—the magnetization reaches saturation. For pure iron this means a flux of about 21 500 lines per cm.$^2$

The behaviour of iron when placed in a magnetic field is explained if each atom has an electric spin. It is known that electricity is part of the constitution of the atoms, and if this electricity is rotating in the atoms then each of them is equivalent to a small coil, carrying a current. This coil will try to set itself so that the flux through it due to its own current is in the same direction as the flux in which it finds itself. If there is no flux due to outside currents the guiding flux for each atom is the flux from neighbouring atoms. The atoms will string themselves on each other's lines of flux and so form local closed loops or chains. In this arrangement each loop will be like a toroid and will produce no outside flux; there will be no indication of a magnetic field outside the iron. Things will be different, however, if a field is provided in the iron, by a winding carrying a current for instance. If the magneto-motive-force is small only those atoms will turn round which are not strongly held by their neighbours. Others will only turn slightly, and as the m.m.f. is increased the effect will increase in proportion.

Each atom that turns moves so that its own m.m.f. adds to the m.m.f. of the external current. Before they turned the m.m.f.'s of the atoms were in all directions at random; the turning of the atoms introduces some order and the flux in the iron is increased. When the flux from outside is strong enough to pull the closed chains open and make all the ampere-turns in the chain add their share, there is a considerable increase in the general flux. This is the stage in the curve $AO$, Fig. 59, where it is steepest. All the closed chains are not pulled open together; some are more obstinate and there is in consequence no sharp corner on the curve. When all the atomic chains have been opened there is not much more to do except to pull all the atoms round to lie exactly in the same direction. This requires a strong external field, as the neighbours of each atom tend by their fluxes to pull it round somewhat from exact alignment with the external field. The last stage, therefore, is only completed in quite strong fields. However strong this applied field may be there is a limit to the share which the iron atoms can contribute to the flux—this is reached when the atomic currents are all flowing the same way round and the flux due to each is in the same direction as the applied flux. The flux from any atom is then taken up and passed on through the next in line like the flux of the coils in Fig. 50. Lines of atoms with their currents linked in this way are exactly similar to the solenoid of Fig. 51. The flux density in such a solenoidal arrangement is

$$4\pi n_1 I/10$$

if $n_1$ = number of atoms per cm. in line and

$I$ = current in amperes in each.

If the area round which the small current $I$ circulates is $a$ cm.$^2$, the number of lines contributed to the total flux is $4\pi n_1 Ia/10$.

If in each square centimetre of cross-section of the iron there are $n_2$ such solenoidal strings of atoms, side by side, the total number of lines contributed by the iron atoms per cm.$^2$ is

$$4\pi n_1 n_2 Ia/10 \text{ or } 4\pi N_0 Ia/10$$

if $N_0 = n_1 n_2$ is the number of iron atoms in one cubic centimetre.

This occurs when the iron is saturated and $4\pi N_0 Ia/10$ is the maximum flux density which the iron can contribute.

If the controlling flux is now reduced somewhat the lines of atoms "ease off," as it were, and the alignment is less perfect. The flux contribution of the iron is reduced, but as each atom is linked by lines of flux with those before and behind, it is not allowed to fall completely out of line. The disturbance of the lines is due to the action of the lines alongside, particularly where they end at the surface of the iron and the flux spreads out from the end of the solenoid (see Fig. 51). This gives the residual magnetism $OB$ in Fig. 59 when the original magnetizing field has been removed. A reverse magnetizing field can reduce these remaining orderly lines in the iron to a state of complete confusion again. Then there is again no flux through the iron, but only a tangled arrangement of flux in closed loops. The reverse m.m.f. required to do this is given by $OC$, and is greater the more obstinately the atoms hold their lines. The magnetization in the reverse direction to $D$ and back again to $A$ via $E$ and $F$ is easily explained in the same way. At any stage the atoms are only partly in order. Some of them cancel out the flux of others so that only $N$ atoms are effective. The flux density is then $4\pi NIa/10$. $Ia/10$ is a fixed quantity for each atom and $N_0$ is the total number of atoms in one cubic centimetre. Hence $N_0Ia/10$ is a constant quantity which can be found by experiment; it is the maximum flux density due to the iron divided by $4\pi$.

When the iron has its atoms aligned in this manner it is said to be *magnetized*. The field which produces the alignment is known as the *magnetizing field*, and the product $NIa/10$ is called the *intensity of magnetization*, and is usually denoted by $J$.

When $N$ atoms per cm.$^3$ are aligned and the fluxes of the rest cancel out in disorder, the magnetization $J$ of the iron is measured by $NIa/10$, which is $1/4\pi$ times the m.m.f. per cm.$^3$ due to the orderly atoms. If a piece of iron in this state is placed in a magnetic field of $H$ lines per cm.$^2$ with the direction of its magnetization perpendicular to the direction of the flux in the field, each atom will be acted on by a torque equal to $IaH/10$ as already found, so that the couple on the piece of iron which tends to turn it to lie along the direction of the field is $NIaH/10$ dyne-cm. per cm.$^3 = HJ$, i.e.

$$\begin{pmatrix}\text{Couple}\\ \text{per cm.}^3\end{pmatrix} = \begin{pmatrix}\text{Flux density of}\\ \text{applied field}\end{pmatrix} \times \begin{pmatrix}\text{Intensity of}\\ \text{magnetization}\end{pmatrix}$$

Couple $= HJ$ dyne-cm. per cm.$^3$ when $H$ and $J$ are at right-angles. $J$ can be measured if the couple in a known field $H$ is measured, but $N$, $I$, or $a$ cannot be measured separately.

The flux density in the iron due to the m.m.f. of the atomic currents is $4\pi J$ lines per cm.$^2$ If there is an applied field $H$ lines per cm.$^2$ the total flux density is $B = H + 4\pi J$.

Though only iron has been discussed so far, there are two other metals which are magnetic. They are nickel and cobalt. In addition alloys containing one or more of the three magnetic metals may also be magnetic, as well as a special group of alloys, known as Heussler, containing no magnetic metal. The magnetic qualities of these metals and their alloys vary enormously from one another. They differ in (*a*) the flux density of the external field required to approach saturation, i.e. to get over the upper bend on the curve in Fig. 59 towards $A$; (*b*) the residual magnetization $OB$ left after being magnetized to $A$; (*c*) the coercive force $OC$, i.e. the reverse flux density of the external field required to demagnetize the material. The range met with is shown in the following table.

TABLE V.

MAGNETIC PROPERTIES

| | Saturation $J$ | Residual $J$ | Coercive Force | Hysteresis (ergs per cycle) |
|---|---|---|---|---|
| Swedish iron | 1 700 | 1 350 | 0·76 | 7 300 |
| Very mild steel, annealed | 1 704 | 1 400 | 0·37 | 3 820 |
| Steel | 1 580 | 800 | 16·5 | 98 000 |
| Steel, hardened | 1 400 | 700 | 52 | 150 000 |
| Cast iron, unannealed | 1 306 | | 11·4 | 35 500 |
| Cast iron, annealed | 1 330 | 320 | 4·6 | 14 500 |
| Tungsten steel, quenched | 1 230 | 850 | 70 | 260 000 |
| Tungsten steel, not hardened | 1 500 | 900 | 26 | 115 000 |
| Cobalt chrome steel | | | 230 | |
| Silicon steel (stalloy, lohys, etc.) | 1 600 | 1 300 | 0·6 | 1 600 to 3 000 |
| Nickel | 479 | 284 | 7·5 | |
| Cobalt | 650 | 200 | 7·8 | 19 000 |

The intensity of magnetization which is produced by a given magnetizing flux density varies greatly with the material tested. The ratio $J/H$ is called the *susceptibility* $k$. It is not a constant by any means, but rises to a maximum and tends to zero as saturation is approached. Fig. 60 shows curves of susceptibility for soft iron, nickel, and tungsten steel. It is, of course, most economical for any purpose, to use material which has a high susceptibility in the region of that flux density in which it is to be employed, and if the susceptibility is at a maximum it may, for small changes of $H$, be treated as a constant without any appreciable error.

The ratio of the total flux density obtained to the flux density applied to produce it is called the *permeability*. The

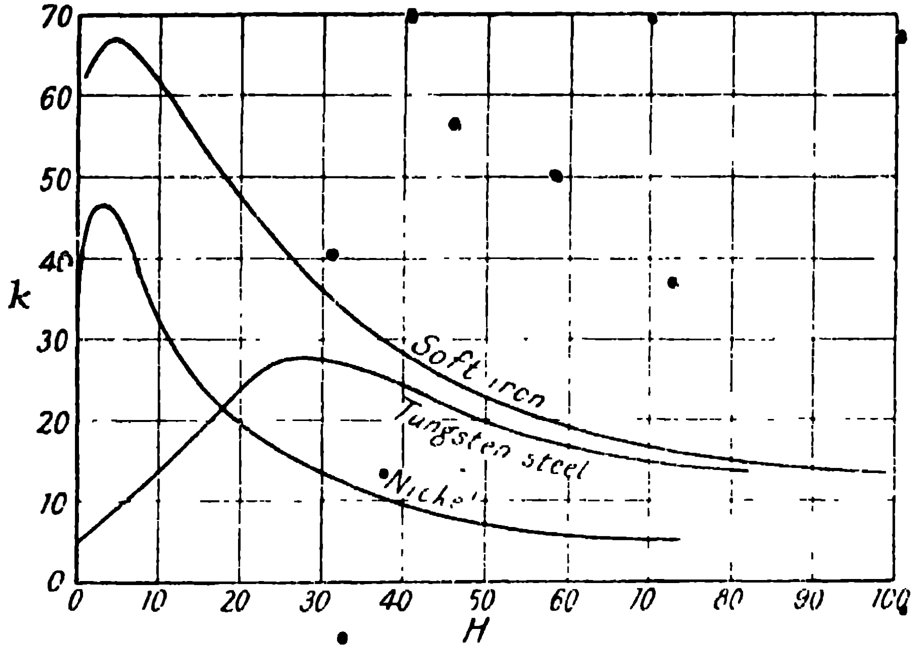


FIG. 60. SUSCEPTIBILITY CURVES
($k = J/H$ for various values of $H$)

Greek letter $\mu$ (pronounced mew) is used for this ratio. The total flux density $B$ is the sum of the applied flux density $H$, and that contributed by the local magnetization $J$, which is $4\pi J$. That is

$$B = H + 4\pi J$$

$$\mu = \frac{B}{H} = 1 + 4\pi\frac{J}{H} = 1 + 4\pi k$$

As $4\pi k$ is much greater than 1, $\mu$ is practically equal to $4\pi k$, and the curves of susceptibility in Fig. 60 can be taken as giving the value of $\mu/4\pi$. The maximum value of the susceptibility (or permeability) occurs at a field strength about equal to the coercive force.

6. **Magnetic Circuit (Including Iron).** The results of the last section may now be applied to the estimation of the flux in a magnetic circuit containing iron.

The magnetic Ohm's law for a circuit containing no magnetic material was considered in Section 2 of this chapter

(p. 85). It may now be extended to the case of a circuit in which iron forms a part. Consider a toroid wound on an iron core. Actually, as has been seen, the total m.m.f. available for providing flux has been increased by the presence of the iron. It is, however, far more convenient to regard the reluctance of the path as having been reduced instead of the m.m.f. increased. Either idea can lead to the same result in the equation

$$\text{m.m.f.} = \text{flux} \times \text{reluctance}$$

Use is made of the expression $B = \mu H$. $H$ is given as before by $(4\pi/10)\ (IN/l)$; the total flux

$$\Phi = Ba = \mu Ha$$
$$= \mu \frac{4\pi}{10} \cdot \frac{IN}{l} a$$

which can be re-arranged as

$$\frac{4\pi}{10} IN = \Phi \frac{l}{\mu a}$$

Here $l/\mu a$ is evidently the reluctance of the iron core of the toroid.

As will be realized from what has been said previously, the reluctance of magnetic materials, being inversely proportional to the permeability, depends to a large extent on the flux density. It is by no means as constant as electric resistance, and this must always be borne in mind when using the "magnetic Ohm's law." The reluctance of air-gaps and of non-magnetic materials is however quite constant; it is simply the length of the air-gap (in cm.) divided by its cross-sectional area (in cm.$^2$).

The form of the expression for the reluctance may be compared with that for resistance

$$R = \rho \frac{l}{a} = \frac{l}{\sigma a}$$

where $\rho$ (Greek rho) is the resistivity of the material and $\sigma$ (Greek sigma) is its conductivity For the reluctance

$$R = \frac{l}{\mu a}$$

Permeability in the magnetic circuit thus corresponds to conductivity in the electric circuit.

Two assumptions have been made in obtaining this expression.

(i) It has been assumed that the diameter of the cross-section is small compared with that of the toroid, so that each line of flux is sensibly of the same length and the lines are uniformly distributed across the cross-section.

(ii) It has been assumed that there is no leakage flux; that is, that all the lines thread all the turns of wire. Actually there is always leakage which may be quite considerable if non-magnetic materials form part of the circuit. The "insulation" of a magnetic circuit is not nearly so perfect as that of an electric circuit.

With these limitations understood, the magnetic circuit idea can be used to estimate the ampere-turns required to produce required fluxes (generally in air-gaps) in more complicated cases. Suppose, for instance, that a saw cut is made in the iron ring, so that a short air-gap of length $l_g$ is made in the magnetic circuit. The reluctance of the air-gap is $l_g/a$, that of the iron is $l/\mu a$ ($l$ is still $\pi d$ very nearly), and the two so that

$$\frac{4\pi}{10} NI = \Phi \left[ \frac{l}{\mu a} + \frac{l_g}{a} \right]$$

Here again an assumption has been made that there is no "fringing" at the air-gap; i.e. that the lines go straight across the air-gap instead of bulging out at the edges as they actually do.

One most important difference between the electric and magnetic circuits must be borne in mind. In the electric circuit there is an actual flow (of electricity), and a continuous expenditure of energy is necessary, because of the resistance of the circuit, to maintain the flow. In the magnetic circuit there is no actual flow and no expenditure of energy is necessary to *maintain* the magnetism. It is true that energy is expended in the electric circuit providing the magneto-motive-force, but this has nothing to do with the magnetism. It arrives only from the resistance of the electric circuit. If the resistance of the circuit were zero no power at all would be required.

Not that magnetism is wholly unassociated with energy. Energy is required to *establish* the magnetic field, and this energy is stored in the field and is available when the field collapses. In this way the magnetic field may be compared with the electric field of the last chapter. The energy stored in the electric field of a condenser was seen to be $\frac{1}{2}CV^2$; the energy stored in the magnetic field of an electric circuit will (in the next chapter) be seen to be $\frac{1}{2}LI^2$, where $L$ is a property of the electric circuit known as its *inductance.*

It must always be remembered also that lines of flux have no actual existence. They are merely a most useful conception of value in obtaining a mental picture of electric or of magnetic fields, and of estimating quantitatively the effects that are produced at different points in the fields. They merely represent in a simple way something which does exist in the field.

7. **Permanent Magnets.** A permanent magnet is a piece of steel which has been magnetized up to Stage $A$ on the curve of magnetization of Fig. 59, and then brought to Stage $B$ by removal of the magnetizing flux. In order that the residual flux shall remain at a constant value, steel of high coercive force is required. The residual magnetization may be reduced by (*a*) a demagnetizing field, (*b*) mechanical shocks, (*c*) variations of temperature. It is found that a steel which is not seriously demagnetized by one of these actions also resists the action of the other two. Mechanical shocks and heating set the atoms vibrating and so tend to produce disorder in their ranks. It has also been found by experience that a proportion of the residual magnetism is very lightly held and easily lost, but the remainder is difficult to remove. The method used to produce a magnet of great permanence is to choose first a material of high coercive force. Next, the shaped piece of steel is magnetized past the upper bend on the curve of magnetization, and then after removal from the magnetizing field it is put in boiling water a few times to shake out the magnetization, which is loosely held, and to keep only the intensity of magnetization, which is not easily lost. Permanent magnets were originally made of high carbon steel, i.e. tool steel, with a coercive force of from 20 to 40 lines per cm.$^2$ Later tungsten steel was used, having a coercive force of 70–80 lines per cm.$^2$, when properly treated, along with a reasonably high residual magnetization. Cobalt chrome steel is now used

as it has a much higher coercive force still. When the composition of the alloy and the heat treatment are exact the coercive force may be over 200 lines per cm.$^2$ Naturally, with atoms which are so difficult to turn, the flux density required to magnetize this material is correspondingly high, but only needs to be applied once in the lifetime of each magnet. Permanent magnets are generally of two forms, either straight bars, needles, etc., or bent round so that their ends are brought near one another. The principal use of the first shape is as a compass needle, the usefulness of which depends upon the fact that if it is freely pivoted at its centre it will turn so as to set with its direction of magnetization along the direction of the flux in which it is placed. This was the only magnetic property known until about a hundred years ago.

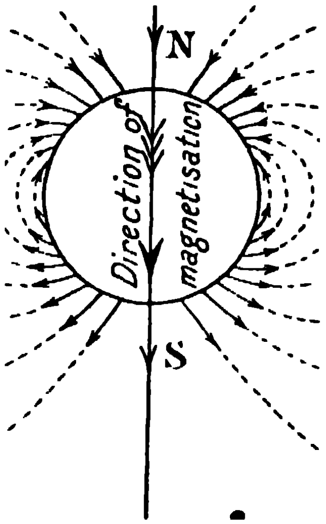


FIG. 61. MAGNETIC FLUX ROUND THE EARTH

The Earth is a permanent magnet with its direction of magnetization nearly agreeing with its axis of rotation. The lines of flux at the surface thus run very nearly from South to North. The lines emerge in the Southern hemisphere and enter the Earth again in the Northern hemisphere. The lines are inclined to the horizontal where they enter and leave, and this angle of inclination is called the *magnetic dip*. Very often the horizontal component only of the Earth's field is of interest; it is this component which directs a compass needle.

The horizontal direction of the lines deviates from true geographical North and South by an amount which varies from place to place and also gradually changes with time. This error of the compass is called the *declination*. The lines of the Earth's magnetic field, in a plane through the centre of the Earth, are shown in Fig. 61. The variation of the dip is shown as the varying inclination of the lines of flux where they meet the surface.

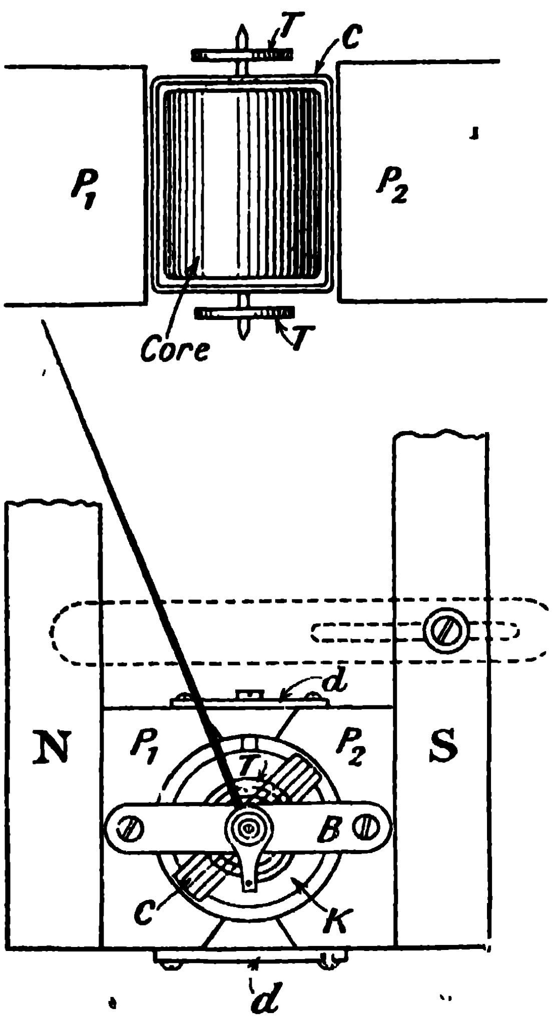


Fig. 62

Moving Coil Instrument

The other type of permanent magnet is used where a magnetic flux of constant strength is required, as in moving coil instruments. The lines of flux pass round in the body of the magnet and complete the loop across the gap between the two ends. The places where the lines of the flux leave the metal and enter the air are called the *poles* of the magnet. "Pole' is a word meaning "end." These are the ends of the orderly strings of atoms in the magnet. The lines of flux spread out from one pole and crowd in again at the other forming closed loops through the body of the magnet. If the gap were completely filled with iron so that the lines were carried across by the m.m.f. of the atoms in it, there would be no spreading out of the lines. If a coil were wound on this iron in the gap and a demagnetizing current passed through it which had the same m.m.f. as the atoms of iron in the gap, things would be the same as if the iron in the gap were taken away. A gap in a magnetized iron ring therefore behaves as a demagnetizing m.m.f., and has a tendency to demagnetize the magnet. The gap should therefore be as small as possible. The poles of magnets for instruments are for this reason provided with pole-pieces of soft iron, and only sufficient room is left to allow the moving parts freedom of motion. To adjust the flux density in the gap to the required value a piece of soft iron may be laid across from pole to pole. This robs the gap of some of the flux carried by the magnet, and by varying the position of this magnetic shunt the flux in the gap is adjusted to the desired value. The shunt is shown dotted in Fig. 62.

8. **Moving Coil Instruments.** The torque exerted on a coil carrying a current in a magnetic field is the principle most widely used in the construction of instruments for measuring direct currents. If the magnetic flux is kept constant the couple $G$ is simply proportional to the current in the coil. If the torque is balanced by a spring the deflection of the spring will be proportional to the torque and thus to the current. The constancy of the instrument, which is its accuracy, depends on the constancy of the magnetic field and of the elasticity of the spring. Permanent magnets are suitably shaped pieces of one of the magnetic metals or alloys which have a magnetic flux permanently linked with them. With suitable material and proper methods of magnetization this flux is remarkably constant, particularly when cobalt steel is used. Hard rolled bronze is used for the springs and when properly treated its elasticity remains very constant.

Fig. 62 shows the construction of a typical moving coil instrument. $N$, $S$ are the ends of a horse-shoe shaped permanent magnet which provides a flux of constant strength. Iron pole-pieces, $P_1$ and $P_2$, convey the flux to the space in which the moving coil $C$ rotates. The faces of the pole-pieces are bored out to make a cylindrical hole, and this is kept true by brass or other non-magnetic distance-pieces $d$ at each side. Fixed to these is the soft iron core $K$, which is thus held concentric with the pole faces. The coil is pivoted so as to turn on the axis of the cylindrical cavity. This arrangement enables a small gap to be used and yet gives free movement to the coil. Current is led into and out of the coil by the spiral springs $T$ at top and bottom. These springs also provide the opposing torque on the coil when it is deflected, and the amount of rotation of the coil is measured by a pointer and scale. The flux provided by the magnet passes across by the shortest path from the pole-pieces to the core, so that the lines of flux are radial and therefore they lie in the plane of the coil whatever its deflection. The flux density is constant in all positions of the coil; thus the deflecting torque $HnAI/10 = KI$ depends only on the current.

The opposing torque due to the springs is proportional to the deflection $\theta$ and so is equal to $s\theta$, if $s$ is the stiffness of the springs. When the coil comes to rest therefore $s\theta = KI$ or

$$I = \frac{s}{K}\theta$$

the current is proportional to the deflection and the scale divisions are equal for equal increases of current.

This type of current measuring instrument is the one most generally used for d.c. measurements. Instruments can easily be made with a coil of 10 ohms resistance to read down to 2 microamperes per mm. division of the scale, and by shunts and series resistances any higher range of amperes and volts may be obtained as explained in the preceding chapter.

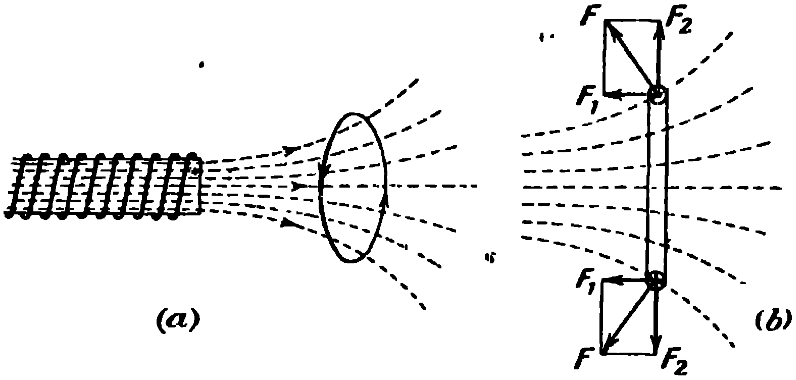


FIG. 63. FORCE ON COIL IN DIVERGING FIELD

Greater sensitiveness to small currents may be obtained by suspending the coil by fine bronze strip instead of using pivots and spiral springs. Currents down to one-hundredth of a microampere may thus be measured. These instruments are used in detecting the balance when using a Wheatstone bridge or a potentiometer, and are known as *galvanometers*.

9. **Moving Iron Instruments.** Consider a circular coil carrying a current and situated in the diverging field from the end of a solenoid or bar magnet (Fig. 63 (*a*) ). The coil is supposed already to have turned so that the flux due to its own current is in line with the flux of the solenoid, and the directions of the flux and current may be as indicated by the arrowheads. It will be shown that there is a further force on the coil tending to attract it towards the solenoid. This force arises because of the spreading out of the lines of flux.

For considering the elevation of Fig. 63 (*b*), the force $F$ on each element of length of the coil, which must be at right-angles both to the field and to the conductor, can be resolved into two components $F_1$ parallel to the axis of the solenoid, and $F_2$ perpendicular to the axis. All the forces $F_2$ cancel out as far as movement of the coil bodily is concerned, but the forces $F_1$ add up and draw the coil towards the solenoid.

The coil moves in fact to a place of greater flux density so as to embrace more lines of flux, and this is a general rule for all movements of current-carrying coils in magnetic fields. Even the forces $F_2$ would, if they could, stretch out the coil so as to make it embrace more lines of flux.

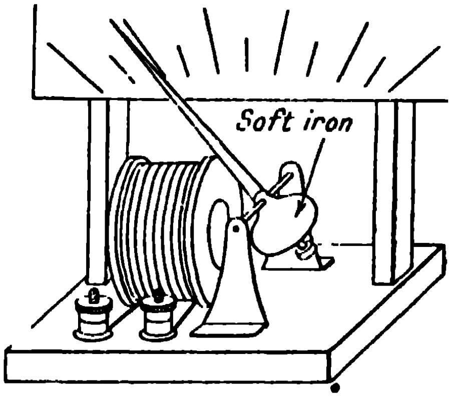


FIG. 64. SIMPLE MOVING IRON AMMETER

Now suppose the coil to be replaced by a piece of iron. The atomic currents will first turn so that their m.m.f.'s are in line with those of the flux and then will try to move so as to embrace more flux. Each atom behaves in fact as the coil, and as a result the iron is attracted towards the end of the solenoid. If the iron were already completely in the uniform field inside the solenoid it would experience no force.

Iron in a magnetic field is thus magnetized and also drawn towards the stronger parts of the field. This attraction can be used to measure a current. Instead of using a magnet to produce the magnetic field, a coil is used through which passes the current to be measured. A piece of iron is supported by a pivoted axle which also carries a pointer (Fig. 64). A current in the coil magnetizes the iron and draws it into the stronger part of the field. The force of attraction increases with the current. Further, if the current is reversed there is still the same attraction so that the instrument may be used for a.c. measurements. The distribution of the divisions on the scale of the instrument may be varied by varying the shape and position of the moving iron. Fixed pieces of iron, also magnetized by the current to be measured, are sometimes used in the construction. They affect the flux density and the divergence so that the required sensitiveness and length of scale may be obtained.

10. **Magnetic Pull.** The pull which can be exerted by a magnet may be very great, and is used in many ways practically. Consider two parallel flat faces of iron with a flux density of $B$ lines per cm.$^2$ across the gap. This means practically an intensity of magnetization $J = B/4\pi$ in each. It can be shown, and it is easily understood, that half the flux in the gap is due to the m.m.f. from one side, and half to the m.m.f. from the other. The flux due to one of the iron faces does not produce any pull on the iron on that side, consequently the pull is entirely caused by the flux due to the other side of the gap. This flux has density $B/2$, and the result is arrived at that the pull per cm.$^2$ drawing one face towards the other is $\frac{1}{2}BJ = B^2/8\pi$ dynes per cm.$^2$ To produce a large pull, as in a magnet used for lifting or in a magnetic chuck, as great a flux density is required as can be arranged for, and as large an area as possible. Now the flux is produced by the m.m.f. of a coil carrying a current, and it is known that the average magnetizing flux density round any loop linked with the coil is $4\pi$ times the ampere-turns per centimetre length of the loop. For a given number of ampere-turns the length of path round the loops of flux should be as short as possible in order to get a strong magnetizing flux density and therefore a high density of magnetization $J$. The area of the iron should be also as large as possible. These ideas lead to the form of magnet shown in Fig. 65 in section (the lines of the flux are shown dotted).

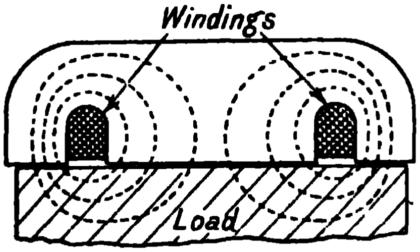


FIG. 65. LIFTING MAGNET

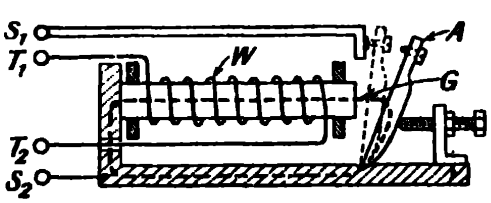


FIG. 66. TELEPHONE RELAY

Another type of device in which this pull is used is the telephone relay shown in Fig. 66. The movement of the armature $A$ under the pull at the gap $G$ is caused by the current in the winding $W$ connected to the terminals $T_1$, $T_2$, and makes and/or breaks the contact which connects or disconnects the terminals $S_1$ and $S_2$. The flux passes from the fixed to the moving part of the iron circuit at the knife edge on which the armature rests, and so to the gap between the core and the armature. With a given gap between the core and the armature and a certain pull required against gravity, a certain number of ampere-turns is required to produce enough flux to operate the relay. These ampere-turns can be provided by winding a suitable number of turns on the core according to the size of the current available for the relay.

A third type of apparatus in which the pull of a magnet across a gap is employed is illustrated by the telephone receiver shown in Fig. 67. A permanent magnet $M$ produces a steady flux between the pole pieces $P$ and the soft iron diaphragm $D$. Round the pole-pieces many turns of fine wire are wound. The alternating speech current passes round these windings, and alternately increases and decreases the flux passing between the pole-pieces and the diaphragm. This produces a fluctuating pull on the diaphragm and sets it in motion, the movement corresponding to the changes of current. Thus the air is moved and sound waves carry the speech to the ear of the listener. If the steady flux density is $B$ and this is increased at a certain instant by an amount $b$ due to the speech a.c., the pull per square centimetre is increased from $B^2/8\pi$ dynes to $(B + b)^2/8\pi$ dynes. The increase is

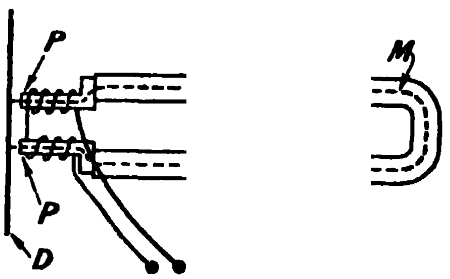


FIG. 67. TELEPHONE RECEIVER

$$\left(\frac{2Bb}{8\pi} + \frac{b^2}{8\pi}\right) \text{ dynes per cm.}^2$$

$b^2$ is negligible compared with $2Bb$, so that the change in pull is $Bb/4\pi$ dynes very nearly.

If the flux density is diminished by $b$ then the pull per cm.$^2$ is diminished by $Bb/4\pi$ dynes. The pull on the diaphragm varies in this way in proportion to the a.c. speech current and the sound heard is intelligible. Without the permanent magnet to produce the steady flux $B$ the flux density would vary from $+ b$ to $- b$ and the pull would increase by $b^2/8\pi$ in each case. The motion of the diaphragm would be inward whether the speech current was in one direction or the other, and the sound heard would not be intelligible, and would be very faint. In addition to faithful reproduction the *polarizing magnet*, as the permanent magnet is called in this and similar cases, increases the sensitiveness in the ratio of the pull in the two cases, i.e. $2B/b$ times.

11. **Pole Strength and the Magnetometer.** It can very well be imagined that the flux lines from each end of a very long solenoid (or bar magnet) spread out uniformly in all directions in the same way as the electric flux lines from an isolated charge (Fig. 46 (*b*) ), and that if two very long solenoids (or bar magnets) are placed end to end, but with the ends some distance apart, the flux lines will have the patterns shown in Figs. 46 (*c*) and (*d*) for the electric flux lines between like and unlike charges respectively, according to whether the m.m.f.'s of the solenoids are in the same or in opposite directions.

It can, in fact, be imagined that at the end of the solenoid or bar magnet there is a "magnetic charge" which causes the magnetic flux in the same way as an electric charge causes the electric flux, and mathematically there is no objection, and the whole theory of electromagnetics is usually built up on this assumption. The imaginary magnetic charge is usually called a *magnetic pole*, and it is supposed to have a certain strength and sign. The inverse square law applies to magnetic poles, so that if $m_1$ is the strength of one and $m_2$ that of the other, the force between the two when $r$ cm. apart is $m_1m_2/r^2$ dynes. *Unit pole strength* is defined from this equation. N-poles are those from which the lines of flux proceed and S-poles those to which they proceed. Field strength is defined by the force exerted on a unit N-pole.

If a sphere of radius $r$ is drawn round the end of a very long solenoid, $r$ being large compared with the diameter of the solenoid but small compared with its length $l$, the area of the sphere is $4\pi r$ and the flux density $H$ across it may be considered uniform and equal to $\Phi/4\pi r^2$, where $\Phi$ is the total flux from the solenoid. But

$$\Phi = \frac{4\pi}{10} \cdot \frac{NI}{l} \cdot a$$

and hence $$H = \frac{\frac{4\pi}{10} \cdot \frac{NI}{l} \cdot a}{4\pi r^2} = \frac{NIa}{10l} \cdot \frac{1}{r^2}.$$

But in terms of the pole strength $m$ of the solenoid this is equal to $m/r^2$.

Hence $$m = NIa/10l \text{ and } \Phi = 4\pi m$$

i.e. from each unit pole there are $4\pi$ lines of flux.

If a long iron rod of cross sectional area $a_1$ is placed in the solenoid, it is magnetized, say, with an intensity $J$, the total flux from its end is $4\pi Ja_1$, and the field strength at radius $r$ is

$$F = \frac{Ja_1}{r^2}$$

Use is made of these expressions in experiments with the magnetometer.

A *magnetometer* is an arrangement of a magnetized needle suspended so as to be free to rotate in a horizontal plane, with some means of reading the angle through which it turns. The reading may be made by a pointer on a scale divided in degrees or by a mirror and scale using a beam of light as a pointer. When there is no other magnetic flux the needle will set itself along the direction of the horizontal component $H_e$ of the earth's magnetic flux. If another flux whose density it is desired to measure is arranged to be at right-angles to the direction of $H_e$, the resultant flux $R$ will be inclined to both as in Fig. 68 (*a*). This figure shows two fluxes at right-angles, one $H_e$ lines per cm.$^2$ the other $F$ lines per cm.$^2$ The resultant flux has a magnitude and direction $R$, and it is along $R$ that the needle will set, and $\theta$ the angle between $H_e$ and $R$ can be read. Clearly,

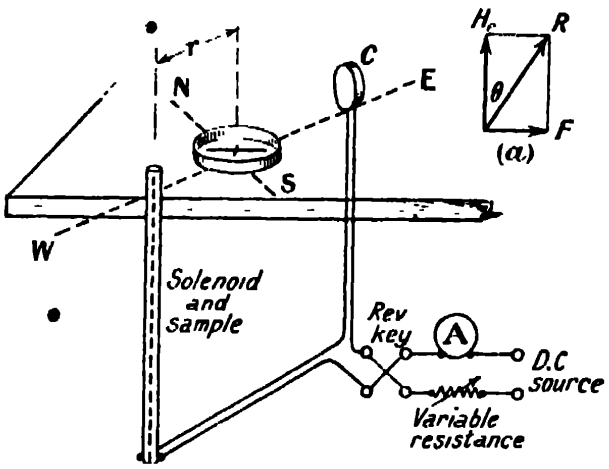


FIG. 68. MAGNETOMETER

$$F = H_e \tan\theta$$

To determine $H_e$, a solenoid is arranged vertically as in Fig. 68, with one end due west of the magnetometer, and distant $r$ from the centre of the needle. Current is supplied to the solenoid from a d.c. source, and for each current

I the field $F$ at the magnetometer is, as has been shown above,

$$F = \frac{NIa}{10l} \cdot \frac{1}{r^2}$$

(For this experiment the coil $C$ is removed.)

The deflection $\theta$ is noted for various currents $I$ and distances $r$ and for each the earth's field is found from

$$H_e = F/\tan\theta$$

This experiment serves at the same time as a determination of the horizontal component of the earth's field and a verification of the truth of the inverse square law.

The magnetometer can also be used for a more precise determination of the hysteresis loop of Fig. 59 than is possible with the apparatus of Fig. 55. The arrangement is as shown in Fig. 68, the coil $C$ placed due east of the magnetometer being so disposed that, when there is no iron rod in the solenoid, the coil $C$ and the solenoid produce equal and opposite fluxes at the magnetometer, and hence no deflection. When a long iron rod of the material to be tested is put into the solenoid the deflection of the magnetometer is due solely to the magnetization of the rod. The flux density $F$ at the magnetometer due to the magnetization $J$ is, as has been seen above,

$$F = \frac{Ja}{r^2}$$

where $a$ is the cross-sectional area of the rod. $H_e$ has already been determined and the reading $\theta$ gives $J$ since

$$F = \frac{Ja}{r^2} = H_e \tan\theta$$

and

$$J = \frac{H_e \tan\theta}{a} r^2$$

The m.m.f. of the coil is $4\pi IN/10$ and $B = 4\pi J$ (very nearly), so that all the data required for the curve of Fig. 59 can be obtained.

Actually it is more usual to plot the magnetizing flux $H = 4\pi IN/10l$ horizontally instead of the m.m.f., and the curve is then known as the $B$–$H$ curve.

Starting with the rod demagnetized a gradually increasing magnetizing field is applied and the corresponding deflections of the magnetometer are noted. From each value of the current the magnetizing field *H* is calculated and from the tangent of the deflection of the magnetometer, the intensity of magnetization *J*. In this way curves of magnetization can be obtained for any sample in the form of a rod. After reaching the maximum the magnetizing current is gradually reduced to nothing, reversed, increased to the maximum, reduced and reversed so as to complete the cycle.

## CLASS EXPERIMENTS

### ON THE SUBJECT OF CHAPTER III

1. Make iron filings maps of the fields of a circular coil, two parallel co-axial coils, a solenoid without or with an iron core, and a toroid; a bar magnet, a horse-shoe magnet, and a horse-shoe magnet with a shunting iron bar.
2. Map the field of a vertical straight current with a compass needle showing the distortion of the shape of the flux lines by the Earth's field. Find the point at which the Earth's field and the field of the current neutralize each other, and measure the distance *d* from this point to the conductor. Show the current strength *I* divided by *d* is a constant when the current is varied.
3. Using a number of different toroids, show, using the apparatus of Fig. 53, that the flux density depends only on the product $NI/l$. Confirm as far as *N* and *I* are concerned by the apparatus of Fig. 58.
4. Using the apparatus of Fig. 54, show that the force on a conductor is proportional to the product $HIl$.
5. Plot the hysteresis loop of the magnetic circuit of the apparatus of Fig. 58 in the manner described in the text.
6. Use the magnetometer and the apparatus set up as in Fig. 68 to find (i) the horizontal component of the Earth's magnetic field, (ii) the hysteresis loops for rods of annealed soft iron, annealed tool steel and hardened tool steel.
7. Calibrate a moving iron instrument against a moving coil instrument for increasing and decreasing currents, and look for any hysteresis effect.

## EXERCISES

1. Calculate the torque on a coil of rectangular shape 3 cm. × 2·5 cm. and 50 turns in a magnetic field where the flux density parallel to the plane of the coil is 1 500 lines/cm.². The current in the coil is 20 mA. (1 125 dyne–cm.)
2. A bar of copper is carrying a current of 1 500 A. in a magnetic field of 400 lines/cm.² perpendicular to the bar. Calculate the load per ft. on the bar. (4·1 lb. wt.)

3. A solenoid is 26 cm. long and 5 cm. diameter and consists of 1 000 turns of wire. Find the magnetic flux density and the total flux in it, when a current of 1·3 A. flows through the wire. (63 lines/cm.$^2$; 1 240 lines.)

4. If the susceptibility of a magnetic material is 3·5, what is its permeability? (45.)

5. A soft iron ring of cross-section 12·5 cm.$^2$ and radius 10 cm. is wound uniformly with 250 turns of wire. Calculate the flux in the ring when the current in the winding is 0·15 A. The permeability may be assumed to be 650. (6 100 lines.)

6. By how much must the current in Question 5 be increased to produce the same flux if there is an air-gap of 1 mm. in the iron ring? (0·155 A.)

7. If the effective area of a lifting magnet is 1 sq. ft. and the flux density is 1 000 lines/cm.$^2$, what is the maximum weight that the magnet can lift? (83·2 lb.)

## CHAPTER IV

# ELECTROMAGNETICS: THE GENERATOR EFFECT

1. **Electromotive Force.** A study has now been made of electrolysis, the heating produced by a current, the magnetic field which is set up round the line of flow of a current and the motor effect, that is, the force which acts on a conductor carrying a current when it is in a magnetic field. In all these cases a source of current was mentioned but nothing was said, really, about what that source might be or how the electricity was driven round the circuit setting up potential differences. It has been seen that if there is such a source, current flows, setting up p.d.'s according to the resistances and the capacitances in the circuit. There remains now the study of the sources which can be used to produce the current.

It was pointed out when discussing electric charges that a source could be measured by the amount of work it did per unit charge delivered. This, by the law of the conservation of energy, must be the same as the amount of work which the unit charge would do in passing round the circuit. This energy put into the circuit per coulomb is called the *electromotive force* (e.m.f.) of the source and can be measured in volts, for the number of volts is a measure of the energy per coulomb in joules. The difference between e.m.f. and p.d. is this—

*The e.m.f. is the energy put in by the source per unit of charge it supplies, and the p.d. between any two points is the work which can be done by unit charge passing from one point to the other.*

Going back to the analogy of the water pump and tanks in the first chapter, the e.m.f. corresponds to the energy used in driving the pump per cu. ft. of water supplied, and the p.d. corresponds to the pressure head developed in each part of the system of pipes or apparatus connected to the pump. If the output of the pump is throttled so that the flow is reduced and finally stopped, the pressure head at the throttle increases to the maximum the pump can generate. In the electrical case the maximum p.d. the source can produce is its e.m.f. When the throttling does not stop the flow, there will be some frictional loss in the pump, so that the pressure head at the output is less than the maximum. When an electric source is supplying a current the p.d. at the output terminals is below the maximum possible, because of the work done in pushing the electricity through the resistances inside the source. This gives the power relation

$$EI = VI + I^2R$$

$EI$ = e.m.f. in volts × current
= energy input from source per sec.
$VI$ = p.d. at terminals × current
= energy output from terminals per sec.
$I^2R$ = work spent internally per sec.

Dividing this equation throughout by $I$ gives

$$E = V + IR$$

or
$$V = E - IR$$

$E$ the e.m.f. and $R$ the internal resistance of the source are the two quantities which belong to the source and which have to be reckoned with in every application.

An e.m.f. can be put into a circuit by chemical or magnetic means. The chemical effect will be studied in the last chapter. The magnetic production of e.m.f.'s will now be considered.

2. **Magnetic Induction.** In the last chapter it was shown that a movement of electricity—a current—produced, in the space round the line of its movement, a condition of strain of a particular kind. This is called *magnetic flux*, and the amount of the strain is measured by the *flux density* and can be represented by so many lines of flux per cm.$^2$. Movement of electricity produces magnetic flux. Can this be worked backwards, and can magnetic flux produce movement of electricity? It was not till some time after the first action was known that the second or reverse effect was discovered. The problem was solved by Faraday, and the solution is known as *Faraday's laws of induction of currents.*

(*a*) INDUCTION IN COILS. The induction of currents may be studied experimentally with the apparatus of Fig. 53. The phenomenon has in fact already been used to indicate the equality of magnetic fields. In this apparatus the toroid which produces the magnetic field may be called the *primary coil,* and the small coil which is connected to the ballistic galvanometer may be called the *secondary coil*. The ballistic galvanometer should have a long time of swing, long, that is, compared with the time of any of the movements of the secondary coil that are to be made. In that case the throw of the ballistic galvanometer will measure the total charge or quantity of electricity that has passed through its windings.

Experiments should be made as follows—

With a certain current flowing in the primary coil—

(i) The secondary coil, initially with its plane at right-angles to the lines of flux, is turned through a right-angle, so that its plane is finally parallel with the lines of flux. In this way the lines threading the coil, due to the field established by the primary current, have been changed from a maximum to zero. Note the throw of the galvanometer, and note also that the throw is the same whether the secondary coil is turned slowly or quickly. Note also that if the secondary coil is turned through a further right-angle the same throw is obtained, but in the reverse direction.

(ii) Repeat with secondary coils of different numbers of turns and different areas. Note that the throw is directly proportional to the number of turns and to the area.

(iii) Repeat with different primary currents, that is, with different field strengths. Note that the throw is directly proportional to the field strength.

(iv) Repeat the experiment, but instead of turning the secondary coil, take it completely away from the primary. Note that exactly the same results are obtained, and that if the secondary coil is put back into the primary, the same throw is obtained as when it was removed, but in the opposite direction.

(v) With the secondary coil in position embracing the maximum flux, interrupt the primary current by means of the switch. Note that the same throw is produced as would be produced by removing the coil. Note also that on starting the primary current the same throw is produced, but in the opposite direction.

In all these cases, by one means or another, the number of lines of flux, Φ, threading the secondary coils has been changed from Φ to zero, and the quantity, $Q$, of electricity which is caused to flow by this change is found to be proportional to the primary field strength $H$, the area of the secondary coil $A$, and its number of turns $N$. In fact, $Q$ is proportional to the product $HAN$, i.e. to the product $\Phi N$, since $HA$ is evidently $\Phi$. The product $\Phi N$ is termed the flux linkages of the secondary coil, or simply the *linkages*, and may be denoted by $\Lambda$ (Greek capital lambda).

Now the current, $i$, at any instant in the ballistic galvanometer is the rate of change of the quantity flowing in it, which can be written $\dot{Q}$. The current flows because of an *induced* electromotive-force $e$, induced because of the change of linkages $\Lambda$ through the coil. At any instant Ohm's law applies, and $i = e/R$ where $R$ is the resistance of the secondary circuit consisting of the secondary coil and the galvanometer. Thus

$$i = \frac{e}{R} = \dot{Q} = k\dot{\Lambda}$$

where the dot over $Q$ and $\Lambda$ indicates that the rate of change is intended, and $k$ is a constant.

From this,

$$e = kR\dot{\Lambda}$$

which, since $R$ is also a constant, shows that the experiments have established that the e.m.f. induced in a coil is proportional only to the rate of change of flux linkages with the coil. Actually the units are such that $kR = 10^{-8}$ for $e$ to be given in volts.

The direction of the e.m.f. and current is found by *Lenz's Law*, which states that the induced current flows in such a direction as to try to prevent the change which produces it. If, for instance, the flux linkages are being reduced, the induced current will flow in such a direction as to increase the flux linkages, and so on. This law combined with the corkscrew rule, enables the direction of the induced e.m.f. to be found at once. Lenz's law is a direct consequence of the principle of the conservation of energy.

(*b*) INDUCTION IN A STRAIGHT CONDUCTOR. In the last section the phenomenon of induced electromotive forces and currents has been arrived at by independent experiment. But actually the phenomenon is a necessary consequence of the facts established by the experiments of the last chapter.

It was found, for instance, that a conductor of length $l$

carrying a current $I$ in a magnetic field $H$ at right-angles to the conductor experienced a force $HIl/10$ dynes. In Fig. 69 the conductor $AB$ is shown as moving to the right under the action of the force $F$, the current $I$ flowing upwards and the field $H$ being indicated by the dots as upwards through the paper. The field $H$ is maintained constant. These relative directions are found by the left-hand rule (p. 88). Now when the conductor moves, work is done, equal to $Fx$ if $x$ is the distance moved through, and the energy required for this work must come from the source supplying the current $I$; there is nowhere else for it to come from. If $R$ is the resistance of the conductor $AB$ and $V$ the p.d. between $A$ and $B$, while the conductor is stationary $V = RI$ and $VIt = RI^2t$ joules, i.e. the energy supplied by the source in time $t$ is wholly used up in heating the conductor. But if the conductor moves a distance $x$ in the time $t$ and in consequence does work of amount $Fx$ ergs $= Fx10^{-7}$ joules, the energy $VIt$ supplied by the source must also include this work, and in consequence

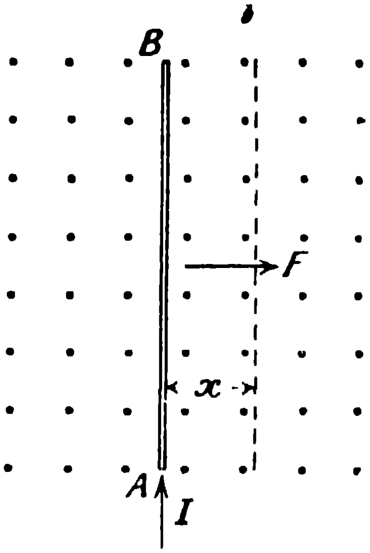


FIG. 69. INDUCTION IN STRAIGHT CONDUCTOR

$$VIt = RI^2t + Fx \times 10^{-7}$$
$$= RI^2t + HIlx \times 10^{-8}$$

Now $lx$ is the area passed over by the conductor and $H$ the flux density. Thus $Hlx$ is the total flux $\Phi$ cut by the conductor, so that

$$VIt = RI^2t + \Phi I \times 10^{-8}$$

whence, dividing throughout by $It$,

$$V = RI + {}^{\Phi} \quad 10^{-8}$$

$$= RI + \dot{\Phi} \times 10^{-8}$$

where $\dot{\Phi}$ as before stands for the rate of change of $\Phi$.

The p.d. at the ends of the wire thus consists of two parts (i) the normal resistance drop $RI$, and (ii) an induced back pressure or e.m.f. $\dot{\Phi} \times 10^{-8}$. Calling this $E$ the equation can be written

$$VI = RI^2 + EI$$

This is the motor equation. $VI$ is the total power supplied, $RI^2$ is the power wasted in heat, and $EI$, which is the product of the back e.m.f. and the current, is the total mechanical power, i.e. that part of the electrical power supplied which is converted into mechanical power.

The e.m.f. $E = \dot{\Phi} \times 10^{-8}$ will be seen to be identical with the induced e.m.f. of the last section; the conductor has only a single "turn" so that $N = 1$ and $\dot{\Phi} = A$. Its direction is clearly such as to produce a current downwards, in conformity with Lenz's law.

Now suppose that the source of the current is removed, but that the circuit is completed, and that the conductor is moved by supplying to it mechanical power. It is reasonable to suppose that the electromotive force is induced in exactly the same way as before, since it depends only on flux lines being cut, and if the movement of the conductor is to the right, a current $I$ will flow downwards from $B$ to $A$. Because of this current in the magnetic field, the conductor will experience a force $F = HIl/10$ dynes acting from right to left, i.e. opposing the motion, and work $Fx$ must be done to move the conductor through a distance $x$ cm. Work must also be done to produce heat, and this is $RI^2t$ joules, where $t$ is the time taken to move a distance $x$ cm. If $V$ is the p.d. established between the terminals $AB$ of the wire, the work done in the circuit external to $AB$ is $VIt$ joules and the work or energy equation is

$$HIlx \times 10^{-8} = VIt + RI^2t$$

which, as before, gives

$$E = V + RI$$

where $E = \dot{\Phi} \times 10^{-8}$.

The wire now acts as a "generator" of electric current. Mechanical power is supplied to the wire and electrical power is produced. Multiplying by $I$ gives

$$EI = VI + RI^2$$

This is the generator equation. $EI$ is the mechanical power supplied to the generator, $RI^2$ is that part of the electrical power produced which is wasted as heat in the generator, and $VI$ is that part of the electrical power produced which is used in the external circuit.

The energy equation gives also, by dividing throughout by $It$,

$$Hl\frac{x}{t} \times 10^{-8} = V + RI$$

from which it is clear that the induced e.m.f. is $Hlv \times 10^{-8}$, where $v$ is written for the velocity of motion of the conductor $x/t$ in cm. per sec.

Thus the two expressions

$$E = \Phi \times 10^{-8} \quad \text{volts}$$
$$E = Hlv \times 10^{-8} \quad \text{volts}$$

are really exactly the same, but sometimes one is more convenient than the other. Either can be written

Induced e.m.f. = Number of lines cut per sec. × $10^{-8}$ volts.

The direction of the induced e.m.f. depends upon Lenz's law as explained above. It can be memorized by the *right*-hand rule. (Compare with the left-hand rule for the motor, p. 88.) The thumb and first and second fingers of the right hand are set mutually at right-angles, and then if the

Thumb points in the direction of the motion
and the first finger points in the direction of the flux (field)
then the second ,, ,, ,, ,, ,, current

It may at first sight be thought that the cases of this section and the last are different. In this section the e.m.f. is said to arise because the conductor cuts lines of flux; in the last section it was said to arise because the flux threading a circuit was changed. But in this case the flux-lines must have been cut by the conductors of the circuit as the linkages were changing, and so the two cases are really the same. Even when the flux change is caused by the establishing and collapsing of a field, the conductors are cut by the lines, which must be imagined to grow out of the (primary) conductors and spread to take up their final position, and to collapse into the (primary) conductors in the same manner.

**3. Alternating e.m.f.'s.** The most important case practically of the production of an e.m.f. by the motion of a conductor in a magnetic field is the generator. In this way mechanical energy is converted into electrical energy which can be transmitted efficiently to the place where it is required. The principle used in every generator of electric energy from mechanical energy is the movement of a conductor across a magnetic field. The e.m.f. generated is $Hlv \times 10^{-8}$ volts, as has been seen earlier in this chapter.

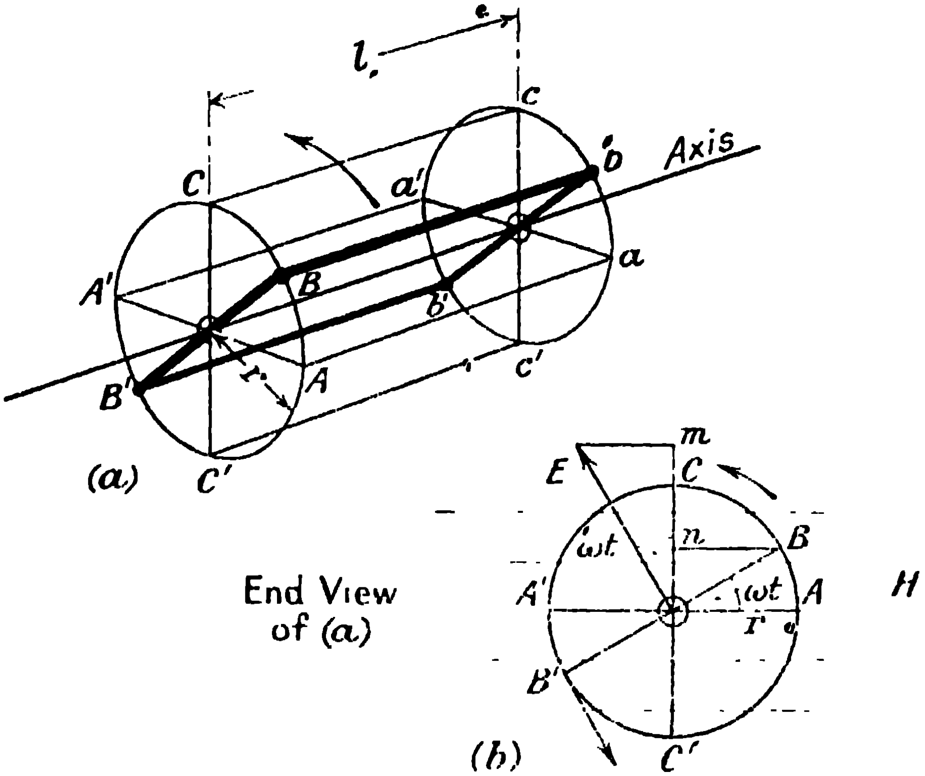


FIG. 70. E.M.F. IN ROTATING COIL

To produce a sufficiently high e.m.f. requires a value of $v$ which is numerically large in cm. per sec. This, clearly, can only be obtained by a rotation. The conductor is in the form of a coil which is generally of rectangular shape and it is rotated in a strong magnetic field. Consider such a coil rotating about an axis parallel to its sides in a uniform magnetic field of flux density $H$ lines per cm.$^2$ Let the length of each side of the coil be $l$ cm. and the distance of the side from the axis be $r$ cm. A diagram of the coil is shown in Fig. 70 (*a*). $AA'a'a$ is the coil when it lies with its plane parallel to the lines of flux. A little later it has turned to the position $BB'b'b$, then to $CC'c'c$, and so on. In the position $AA'$ in Fig. 70 (*b*), which gives an end view of the coil, no flux passes through the coil; as it turns it encloses more and more lines until the position $CC'$ is reached when the flux through the coil is greatest. Then the flux linked decreases until the position $A'A$ is reached with no flux linked. As the coil turns further the flux entering the coil in the reverse direction increases to a maximum and then declines, and has reached zero when the coil is in its original position. The sides of the coil cut the flux and there is therefore an e.m.f. $e$ induced in each of them; $e = Hlv \times 10^{-8}$ volts. The ends of the coil cut no flux and do not contribute to the e.m.f.

In this formula $H$ and $l$ are constant and only $v$ changes. $v$ is the velocity of the side of the coil across the flux. That is, $v$ is the velocity of $Aa$ parallel to $OC$. This is greatest at $A$ and gets less and less until $C$ is reached, where it is nil. If the angle which the coil turns through per sec. is $\omega$, then the angle $AOB$ is $\omega t$ if $t$ is the number of seconds after the coil passes the position $AA'$. The speed of the side of the coil is $OA \times \omega$, it is constant and is always perpendicular to the plane of the coil $BOB'$. If a line $OE$ is drawn as shown perpendicular to $OB$, and if $OE$ rotates with the coil and is made of length $= \omega \times OA = \omega r$, $OE$ represents the velocity of the coil side $B$, and the component of the velocity of the coil side parallel to $OC$ is given by the projection $Om$ of $OE$ on $OC$. The e.m.f. due to the coil side $Bb$ at any instant is given then by

$$e = Hl \times Om \times 10^{-8} \qquad \text{volts}$$

By Lenz's law this e.m.f. will be in the direction $Bb$, for a current in this direction would be acted on by a force opposite in direction to the motion. The e.m.f. in the side $b'B'$ will by the same reasoning be of equal magnitude and in the direction $b'B'$, i.e. added to the e.m.f. in the side $Bb$ to drive a current round the coil. The ends $BB'$ and $bb'$ cut no flux and therefore have no e.m.f. induced in them. The total e.m.f. round this coil of one turn will therefore be

$$e = 2Hl \times Om \times 10^{-8} \qquad \text{volts}$$

The maximum value occurs when $Om = OE = \omega r$ and is

$$E = 2Hl\omega r \times 10^{-8} \qquad \text{volts}$$

But $2lr$ is the area $A$ of the coil and $HA$ is the maximum flux $\Phi$ threading the coil. Thus the maximum e.m.f. is

$$E = \omega\Phi \times 10^{-8}. \qquad \text{volts}$$

and if the length $OE$ is made equal to this, its projection $Om$ will give the e.m.f. at any instant as the coil rotates.

A curve of the e.m.f. plotted against the time can therefore be drawn in exactly the same way as described on p. 53.

The flux linked with the coil is evidently proportional to the projection $On$ of $OB$ on $OC$, and if the length $OB$ is made equal to $HA = \Phi$ the flux at any instant will actually be given by $On$.*

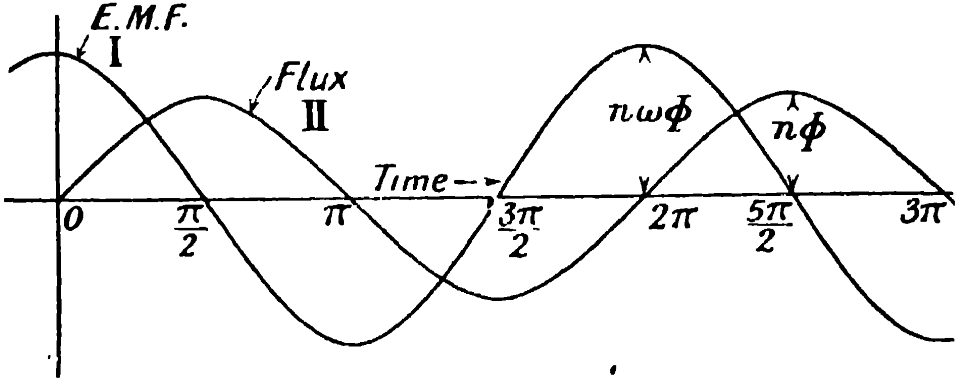


FIG. 71. RELATION OF E.M.F. AND FLUX

If the coil consists of $n$ turns the e.m.f. and flux linkages are $n$ times as great, and $OE = n\omega\Phi \times 10^{-8}$ and $OB = n\Phi$.

The graph of the e.m.f. on a time base is shown in Fig. 71, curve I. There is also shown in curve II the flux linked with the coil. The values of $\omega t$ are marked on the base line. The curves are drawn for 1½ revolutions.

These curves should be compared with those on p. 54. It is seen that they are of the same shape as those assumed for the p.d. from the a.c. source, and that the e.m.f. and flux curves occupy the same relative positions on the time base as the current and charge or p.d. curves for a condenser. This is a natural consequence of the fact that the e.m.f. is given by the rate of change of the flux, and the current is given by

---

$$
\begin{aligned}
{}^{*}\,On &= OB \sin \omega t = r \sin \omega t\\
\phi &= 2On \times lH = 2rlH \sin \omega t = \Phi \sin \omega t\\
e &= \dot{\Phi} \times 10^{-8} = \omega\Phi \cos \omega t \times 10^{-8}
\end{aligned}
$$

the rate of change of the charge. Using the dot symbol to indicate rate of change this is expressed by the equations

$$e = \omega\dot{\Phi} \times 10^{-8}$$

$$q = \omega C v$$

If the terminals of the coil of many turns $n$ are taken to copper rings mounted on the axis, a circuit external to the coil may be supplied, from carbon brushes resting on the rings, with alternating current. This rotating coil is, in a very elementary manner, a description of the principle of the source of electricity known as an *alternating current generator*, or simply, an *alternator*. In the actual machine the coils are wound on a fixed frame and the field, provided by electromagnets, rotates. Iron is used everywhere to increase the flux density, and only a narrow air-gap is left between the rotating field system and the coil windings.

The elementary principle of the d.c. generator or dynamo is the same, but instead of the slip rings a *commutator* is provided. This is an arrangement which, in the simple coil under consideration, simply reverses the terminals of the coil with regard to the external circuit each time the e.m.f. in the coil is reversed. In this way the current is caused always to flow in the same direction. In the actual dynamo the coil system rotates and the field system is stationary, and there are many coils whose combined effect is to give an almost constant e.m.f.

4. **Inductance.** Not only are induced electromotive forces of great importance in sources of electricity, but they are of great importance also in all circuits in which the currents are changing, as, for instance, when currents are started or stopped in d.c. circuits, and, of course, in a.c. circuits. All currents have magnetic flux associated with them and when the current changes the flux changes also, and when the flux changes e.m.f.'s are set up in the conductors which the flux threads, not only in neighbouring conductors but also in the conductors which themselves carry the current. The effects of these induced e.m.f.'s are very much greater in some circuits than in others, but they must be considered in all.

(*a*) MUTUAL INDUCTANCE. Suppose there are two coils, 1 and 2, which are close to each other. For convenience Coil No. 1 may be called the *primary* coil and Coil No. 2 the *secondary* coil. Let a current $I$ flow in the primary coil. Magnetic flux is produced and some of this flux is linked with the secondary coil. As the current rises from zero to its value $I$, the flux spreads out from the primary conductors and some of it cuts some of the $N_2$ turns of the secondary coil, and in consequence an e.m.f. is induced in the secondary coil. Similarly if a current $I_2$ is started in the secondary coil, flux will spread out from the secondary coil and some of it will cut some of the turns of the primary coil and induce in it an e.m.f. Thus any change of current in one coil is accompanied by a change of flux linkages through the other coil and hence an induced e.m.f. in the other coil. The phenomenon is known as *mutual induction.*

Let $\Lambda_{21}$ be the flux linkages through the secondary coil caused by a current of one ampere in the primary coil. If all the flux $\Phi_1$ of the primary passed through all the turns $N_2$ of the secondary, $\Lambda_{21}$ would be equal to $\Phi_1 N_2$, but this can never quite occur. $\Lambda_{21}$ is therefore the sum of the products $\Phi N$ taken throughout the coil a few turns or even one turn at a time. For a primary current $I_1$ amperes the flux linkages with the secondary will be $\Lambda_{21} I_1$ and the e.m.f. produced will be the rate of change of these linkages multiplied by $10^{-8}$, whether caused by relative motion of the coils (which changes $\Lambda_{21}$) or by a change of the current.

In an exactly similar manner let $\Lambda_{12}$ be the flux linkages with the primary coil caused by a current of one ampere in the secondary coil. If the secondary current is $I_2$, the total linkages are $\Lambda_{12} I_2$, and the e.m.f. produced in the primary is the rate of change of $\Lambda_{12} I_2$ multiplied by $10^{-8}$.

Now it can be shown that $\Lambda_{12}$ is equal to $\Lambda_{21}$. It has been seen in the energy equation on p. 119 that, in the case of a single conductor, the work done in moving the conductor through a magnetic field is $HIlx \times 10^{-8}$ which is $\Phi I \times 10^{-8}$. If there were $N$ conductors the work would be $\Phi I N \times 10^{-8}$ and this, by considering each small length of the conductors of a coil, can be extended to a general proposition: the work required in joules to change the flux linkages through a circuit by an amount $\Lambda$ is equal to the product of the current $I$ in amperes through the circuit, the linkages $\Lambda$ and $10^{-8}$.

Suppose now the primary coil is moved right away from the secondary coil, and that the currents $I_1$ and $I_2$ in the coils are maintained constant throughout. The linkages through the primary coil have been changed from $I_2A_{12}$ to zero, and the work done is, by the above proposition, $I_1I_2A_{12} \times 10^{-8}$. Now suppose the secondary coil is brought up to the primary coil so that it again occupies the same relative position. The linkages with the secondary due to the primary current have been increased from zero to $I_1A_{21}$, and the work done in moving the secondary is $I_2I_1A_{21} \times 10^{-8}$. Clearly since the two movements are relatively equal, the work done in each must also be equal. Thus

$$I_1I_2A_{12} \times 10^{-8} = I_2I_1A_{21} \times 10^{-8}$$

and $A_{12} = A_{21} = A$, say.

If then the coils remain stationary $A$ is a constant and the e.m.f.'s are caused in one circuit by a change of current in the other. In fact, $e_1 = (A \times 10^{-8})\dot{i}_2$ and $e_2 = (A \times 10^{-8})\dot{i}_1$ where the dot as before indicates rate of change. The quantity $A \times 10^{-8}$ is called the *mutual inductance* of the two coils (or the *coefficient of mutual induction*). It is always indicated by $M$ and is measured in henries. It is a constant for any relative position of two circuits provided that the flux is wholly in non-magnetic materials. But if iron is present the value of $M$ is no longer a constant, for, as has been seen in the last chapter, the flux produced by a given current is no longer proportional to the current.

The mutual inductance $M$ in henries between two circuits can be defined as above as $10^{-8}$ times the linkages in one circuit produced by a current of one ampere in the other. It can also be defined, since

$$e_1 = M\dot{i}_2 \text{ and } e_2 = M\dot{i}_1$$

as the induced e.m.f. in volts produced in one circuit by a current changing at the rate of one ampere per sec. in the other.

The direction of the induced e.m.f. is always such as to try to undo the change of flux which produces it. Thus when the linkage is increasing, the e.m.f. is in the direction which would send a current to drive a flux in the opposite direction and keep the total flux steady. And when the linked flux is decreasing the induced e.m.f. endeavours to keep the flux at its original value by supplying the loss.

When the current in the primary is alternating, the e.m.f. in the secondary is also alternating and is equal to $M \times$ rate of change of primary current in amperes per sec.

It has been seen that if a current is a simple sine wave alternating current then its rate of change is also a simple sine wave a.c. but a quarter of a cycle earlier in phase, and with an amplitude $\omega$ times the amplitude of the sine wave current. $\omega = 2\pi f$ and $f$ = frequency = number of cycles per sec.

The primary current and secondary e.m.f. curves have the same shape and relative positions as the flux and e.m.f. curves of Fig. 71 and the quantity and current curves of Fig. 40, p. 54. The vector diagram is drawn in Fig. 72, $OI_1 = I_1$ = the maximum value of the primary current, and $OE_2 = E_2 = \omega MI_1$, drawn at right-angles to $OI_1$, leading, is the maximum value of the induced secondary e.m.f. $OI_1$ and $OE_2$ are rotated together at an angular velocity $\omega$, and their projections on the vertical through $O$ give the current and induced e.m.f. curves in the manner of Fig. 39, p. 54.*

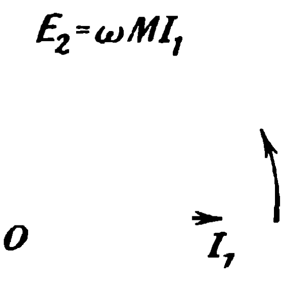


FIG. 72. VECTOR DIAGRAM

Precisely similar considerations apply when there is an alternating current in the secondary coil and an induced e.m.f. in the primary. By taking account of the values of $\omega$ and $M$ the value of the factor $\omega M$ can be made to have a wide range of values, and thus the a.c. volts in the secondary circuit can be adjusted to suit particular needs. This is the principle of the transformer. The secondary circuit is not connected electrically to the primary, and the a.c. voltage in it may be arranged independently from the primary a.c. volts.

(*b*) SELF INDUCTANCE. Considering now a single coil it is easily seen that any turn of it is coupled magnetically by flux lines with the rest of the turns. An e.m.f. must be induced in each turn if this flux linking the turns is varied. The flux changes when the current in the coil varies, and the induced e.m.f. therefore depends on the rate of change of the current

---

* If $i_1 = I_1 \sin \omega t$

$$e_2 = M \frac{di_1}{dt} = \omega MI_1 \cos \omega t$$

in the coil. Just as in the case of two separate coils there is here a coefficient—the self inductance of the coil—which is generally represented by the letter $L$. When the current is varying the induced e.m.f., $e$, is given by the equation

$$e \text{ in volts} = L \times \text{rate of change of current in amperes per sec.}$$

By Lenz's law if the current is increasing the e.m.f. is in the direction to oppose the change, that is, it is backwards; and if the current is decreasing, the e.m.f. tries to boost the current, that is, it is forward. The coefficient $L$, the *self inductance*, depends on the number of turns of wire in the coil and its size and shape, so that a certain coil has a certain self inductance. The unit of self inductance, usually abbreviated to inductance, is the ***henry.*** A coil of $L$ henries has an e.m.f. of $L$ volts induced in it when the current in the coil is changing at the rate of one ampere per sec. Alternatively, as in the case of mutual inductance, the inductance of a circuit is the flux linkages $\times\ 10^{-8}$ produced by a current of one ampere. The e.m.f. hinders or helps the current according as it is increasing or decreasing.

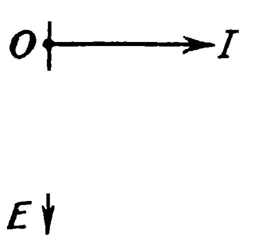


FIG. 73. VECTOR DIAGRAM

The induced e.m.f. is a back e.m.f., and this is expressed by writing a minus sign in front of $L$ in the above expression, which becomes

$$e = -L\dot{i}$$

This has the effect, when $i$ is an alternating current of sine curve form, of causing the vector $OE$ representing the induced e.m.f. to be drawn vertically downwards from $OI$ representing the current (Fig. 73), i.e. at right-angles lagging on the current and therefore diametrically opposite to the applied voltage $OV$. $OE$ is the maximum back e.m.f. and is equal to $\omega LI$ as before. If the coil has no resistance, the source supplying the current has only to overcome this back e.m.f. The p.d. required from the source is consequently represented in the diagram by $OV$; equal to $OE$ and drawn in the opposite direction, i.e. vertically upwards from $OI$ or at right-angles leading the current.

The mutual inductance $M$ between two coils depends on the shape and size of each coil, and the number of turns in each, and; in addition, on how they are placed with respect to one another. By changing the relative position of the two coils the mutual inductance can be varied from a maximum down to zero, when the flux due to the current in one coil is not linked with the other coil at all, or rather when it causes as many linkages through the second coil in one direction as in the other. Turning the second coil further causes the flux in the reverse direction to increase, so that it is easily seen that the mutual inductance between two coils can be positive or negative or zero. The self inductance of a coil, however, is always positive, for the current in a coil always causes magnetic flux to be linked with it and any change of current, changing this flux, will produce an e.m.f. to oppose the change. The self-inductance of a coil can be made very small, if desired, by arranging the winding properly, though it cannot be made to vanish. A simple method of doing this is to double the wire on itself and wind the doubled wire on a bobbin or on any former as shown in Fig. 74. This is known as a *bifilar winding*. Any current in the wire will then pass in opposite directions along two windings of the same length and number of turns lying close together. The magnetic fluxes due to the two windings practically cancel out everywhere except in the narrow gap between them, and as this is very narrow the total flux passing through it is very small. The coils in boxes of standard resistances are generally wound in this way in order that they shall act as simple resistances and not produce e.m.f.'s in the circuit to which they are connected.

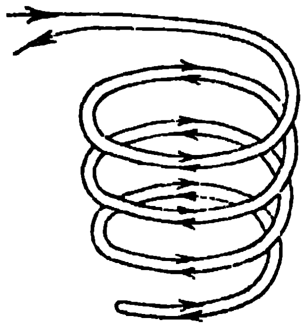


FIG. 74. BIFILAR WINDING

Inductances can be calculated as they depend only on the form of the circuit, but usually the mathematical difficulties are very great. In the case of a toroid, however, the calculation is quite simple, on the assumption that there is no leakage, i.e. that all the flux links all the turns.

The flux $\Phi$ for a current $I$ amperes is given by (p. 85)

$$\Phi = \frac{4\pi}{10} IN \frac{a}{l}$$

and the linkages per ampere $A$ are accordingly given by multiplying this by $N$ and dividing by $I$. Thus

$$A = \frac{4\pi}{10} N^2 \frac{a}{l}$$

and

$$L = \frac{4\pi}{10} N^2 \frac{a}{l} \times 10^{-8}$$

$$= 4\pi N^2 \frac{a}{l} \times 10^{-9} \quad \text{henries}$$

If there is an iron core, the flux and therefore the inductance is $\mu$ times as great.

The mutual inductance between the toroid and the secondary coil of Fig. 53 can also easily be found. Let the subscript 1 refer to the toroid and 2 refer to the secondary. If $a_2$ is the area of the secondary coil and $N_2$ the number of its turns, and if $B$ is the flux density due to a current of one ampere in the toroid, the linkages with the secondary are evidently $Ba_2N_2$.

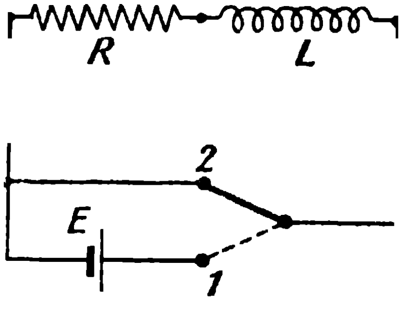


FIG. 75
GROWTH OF CURRENT IN INDUCTIVE CIRCUIT

But

$$B = \frac{\Phi}{a_1 I} = \frac{4\pi}{10} \cdot \frac{N_1}{l}$$

and hence

$$M = 4\pi N_1 N_2 \frac{a_2}{l} \times 10^{-9} \quad \text{henries}$$

5. **Inductance in D.C. Circuits.** (*a*) GROWTH AND DECAY OF CURRENT. The effect of inductance in a circuit when a steady voltage is switched on is to delay the growth of the current to the steady final value given by Ohm's law

$$E = IR$$

Until the current has become steady there is the induced e.m.f. to be reckoned with as well as the e.m.f., $E$, due to the source. A circuit containing a source of constant e.m.f., $E$ volts, total resistance $R$ ohms, and self inductance $L$ henries, is represented in Fig. 75. The e.m.f.'s in the circuit are the applied volts $E$ and the induced e.m.f $-L \times$ (rate of growth of current). As this induced e.m.f. is against the growth of the current and $E$ is in the direction of increasing current, the equation for the current is

$$E - L\dot{I} = RI$$

The final steady value $I_0$ of the current is $E/R$ when $\dot{I} = 0$. Until this occurs,

$$I = \frac{E}{R} - \frac{L}{R}\dot{I} = I_0 - \frac{L}{R}\dot{I}$$

This gives $$\frac{L}{R}\dot{I} = I_0 - I$$

or $$\dot{I} = \frac{R}{L}(I_0 - I)$$

The rate of growth of the current is $R/L$ times the amount the current is short of its final value.

The greater $R/L$ is, the shorter is the time required to reach a steady current. The reciprocal of this quantity, $L/R$, is proportional to the time interval from closing the switch until the current has become effectively steady. $L/R$ is therefore called the *time constant.* The steady value of the current is very nearly attained after five times the time constant in seconds. For example, if $L = 0{\cdot}23$ henry and $R = 50$ ohms, $L/R = 0{\cdot}23/50 = 0{\cdot}0046$ sec. and it will require $5 \times 0{\cdot}0046 = 0{\cdot}023$ sec. for the current to become approximately steady. If the switch in the circuit of the figure is moved over to the contact 2 in such a way that this contact is made before the other is broken, a circuit will be obtained in which a current is flowing, but there is no external source to supply the energy spent in the resistance. The current, however, does not die down at once but only gradually; for as the current dies out there is an e.m.f. induced in the circuit, owing to its inductance, which tends to keep the current going. The equation when $E = 0$ is

$$-L\dot{I} = RI$$

i.e. $$\dot{I} = -\frac{R}{L}I$$

The rate of decrease (minus sign) of the current is $R/L$ times the value of the current. In about five times the time constant

$L/R$ the current reaches its steady value zero. Inductance in a circuit thus prevents the current from changing instantaneously to the value corresponding to the new conditions when the circuit conditions are suddenly changed. If instead of making contact 2 the switch in the circuit just considered merely broke contact, the current would not cease to flow immediately the two parts of the switch separated. The gap inserts a very high resistance in the circuit and the current falls rapidly, that is, $-\dot{I}$ is large. But the induced e.m.f. in the inductance is $-L\dot{I}$ and is therefore large also, and it will give rise to a sufficient p.d. across the high resistance of the gap to drive the current across and sparking occurs. Sparking at the contacts of a switch, if serious, and repeated many times, causes rapid deterioration of the points, and finally a defective contact, or none at all. Various devices are used to prevent or reduce the sparking at relay contacts. One of the simplest is to connect a high resistance $R_s$ across the contact points. The resistance across the gap can never be higher than this value, and the opening of the switch reduces the time constant of the circuit from $L/R$ to $L/(R + R_s)$ seconds, so that the current falls rapidly to a low value. This resistance $R_s$ must be so high that the final steady current is sufficiently small to be negligible in the circuit, or if this is not practicable, the break must be made in two steps. In the first step the resistance $R_s$ is switched into the circuit. This reduces the current rapidly to a low value. The second step of the switch then breaks the circuit. The sparking caused by breaking the small current causes a negligible burning of the contacts. A condenser across the contacts is sometimes used, with or without a resistance in series with it, to reduce the sparking at break. There is no trouble at the "make" as the current starts from zero and only grows gradually after the contact has been established.

(*b*) ENERGY IN MAGNETIC FIELDS. How is it that there is energy in the circuit to keep the current going after the supply of energy from the source has been cut off? Energy must be stored somewhere for there is heating in the spark at the break, often very great heating, and there is heating in all the resistances of the circuit. The answer is that there is energy stored in the magnetic field of a current and this stored energy increases as the current increases. This delays any increase of current, as the source has to supply this energy as well as the energy used in heating the resistance. If a balance sheet of energy for the electric circuit having inductance is drawn up, the amount of energy stored can easily be found. The input of energy per second from the source is $EI$ watts. The expenditure of energy per second in the circuit is $I^2R$ watts. The difference must be the amount stored per second.

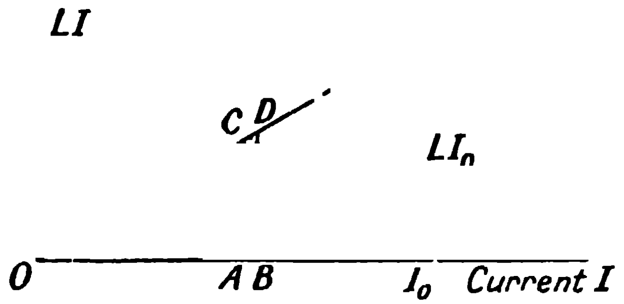


FIG. 76. ENERGY STORED IN INDUCTANCE

The behaviour of the circuit is given by the equation

$$E - L\dot{I} = RI$$

Multiplying each side by the current $I$,

$$EI - LI\dot{I} = RI^2$$

so that the difference between the energy put in and wasted in heat per second is $LI\dot{I}$.

Or, in words,

Energy stored per second = $LI$ times increase of current per second

Consequently the energy stored in any short interval of time is $LI$ times the increase of current in that time. The interval taken must be so short that the current can be assumed as having the average value for the interval. Draw a graph of $LI$ on a base of the current $I$ (Fig. 76). A straight line $OP$ is obtained. At some instant the current has the value $I = OA$ and $AC = LI$. When the current has increased to $OB$ and the value of $LI$ to $BD$ the energy stored is increased by the mean value of $LI \times AB$, that is, the energy stored during this time is equal to the area of the shaded strip. Consequently the energy stored when the current is $I_0$ is the area of the triangle $OPQ$ on the base $OQ$ where $OQ = I_0$. The area of $OPQ$ is equal to $\frac{1}{2}$ base × height, that is, $\frac{1}{2}OQ \times QP = \frac{1}{2}LI \times I$ or $\frac{1}{2}LI^2$ joules.

When it is desired to interrupt the current in a circuit, this energy has to be got rid of in some way. When the current or the inductance is great, special arrangements must be made to deal with it. Very high voltages may be caused in power circuits at the breaking of heavy currents. The design of circuit-breakers, for this reason, is a subject which requires a great deal of study.

The energy stored in the magnetic field can also be expressed in terms of the flux density in the field. For instance, in the toroid

$$B = \frac{4\pi}{10} \cdot \frac{IN}{l}$$

giving

$$I = \frac{10}{4\pi} \cdot \frac{Bl}{N}.$$

and it has been shown above that

$$L = 4\pi N^2 \frac{a}{l} \quad 10^{-}$$

Thus the energy stored in the field

$$= \tfrac{1}{2} L I^2$$

$$= \frac{B^2}{8\pi} a \quad 10^{-7} \text{ joules}$$

But $al$ = the volume of the field, and $10^7$ ergs joule.

Hence,

$$\text{Energy stored per cm.} \quad \frac{B^2}{8\pi} \text{ ergs}$$

From this the pull of a magnet can be found at once. Let the iron surfaces between which the pull is to be found have an area $a$, and let them be separated by means of a force $F$, a distance $x$ cm. The work done is $Fx$ dynes, and this must be equal to the energy stored in the air-gap created, which is $(B^2/8\pi) \times$ volume $= (B^2/8\pi)\,ax$.

$$Fx = \frac{B^2}{8\pi} ax$$

and

$$F = \frac{B^2 a}{8\pi} \quad \text{dynes}$$

a result already obtained by another method on p. 108.

(c) **Energy Wasted in Hysteresis.** In the table on p. 98 there is a column headed "Hysteresis (ergs per cycle)." This gives the amount of energy spent in taking one cubic centimetre of the metal through a complete cycle of magnetization reaching practical saturation in each direction. The figures are only representative of the amounts of energy required for each type of material and are not exact, as so much depends on the preparation of the sample. When a specimen of magnetic material is taken through a cycle of magnetization and the results are plotted in a graph showing $B$ on a base of $H$, the area of the loop is equal to $4\pi \times$ (energy which has been spent in the iron in the cycle). This energy appears as heat in the metal and is the *hysteresis loss* in the cycle. That this area is the wastage of energy can be shown simply if the case considered is that of a ring of the iron wound uniformly with $N$ turns of wire as a toroidal magnetizing coil. Let the mean length round the toroid be $l$ cm., the current in the winding $I$ amperes, the area of section of the toroid $a$ cm.$^2$, and the flux density $B$ lines per cm.$^2$ The flux in the core is $Ba$ lines, and this flux is linked with $N$ turns of the winding. Suppose, owing to a small change of current in the winding, the flux density increases to $(B+b)$ in a time $t$ sec. The linkages with the winding have changed from $NBa$ to $N(B+b)a$ in this time. This gives rise to an e.m.f. opposing the change, i.e. the increase of current, equal to

$$\frac{N(B+b)a - NBa}{t} \times 10^{-8} = \frac{Nab}{t} \times 10^{-8} \text{ volts}$$

That is, the source of current has to expend

$$\frac{NabI\,10^{-8}}{t} \text{ joules per sec.}$$

to maintain the current against this e.m.f. in addition to the expenditure of energy due to the resistance of the circuit. In $t$ sec. therefore the energy spent is $NabI\,10^{-8}$ joules or $NabI/10$ ergs.

The magnetizing flux density is $4\pi NI/10l$ lines per cm.$^2$ $= H$, so the energy spent is $(H/4\pi)al.b$ ergs.

Now $al =$ volume of metal, $b =$ increase in flux.

Hence the energy spent per cm.$^3$ is $Hb/4\pi$ ergs.

*Hb* is the area of the strip shaded in Fig. 77. The total energy used to reach the point *A* on the cycle from the point *F* is the area of all such strips, that is, the area *FAMO*, divided by $4\pi$.

After the point *A* on the magnetizing curve is reached and the field *H* is reduced the value of *B* follows the curve *ADC* to *D*. During this change some of the energy is returned to the circuit by the agency of induced e.m.f.'s. This returned energy is given by the area $AMD/4\pi$. With the reversed field demagnetizing the specimen, energy given by the area $ODC/4\pi$ is spent in reducing the magnetization to zero. Thus in traversing the curve *FADC*, energy is absorbed from the source equal to $1/4\pi$ times the areas *OFAM* and *ODC*, and energy is returned to the source of amount $1/4\pi$ times the area *AMD*, and the net energy taken from the source is $1/4\pi$ times the area *OFADCO*. Exactly similar considerations apply to the other part of the cycle below the axis of *H*. Hence the energy in ergs spent in a cycle is the area inside the loop divided by $4\pi$. This applies to all hysteresis loops. The area may be roughly assessed as that of a parallelogram whose height is twice the maximum *B* and breadth twice *OC*. In order therefore to save this energy loss at every cycle, the material of cores which are subject to alternating magnetization must be made of material with low coercive force *OC*. The material is judged for quality by the energy loss per cycle for a given value of *B*. With 50 cycles a.c. the loss per sec. per $cm.^3$ is 50 times the loss per cycle. That is, the power wasted per unit volume of the iron is proportional to the frequency at a given maximum value for *B*. When the amplitude of the flux is varied, the loss per cycle increases rather faster than in proportion to the increase in *B* maximum.

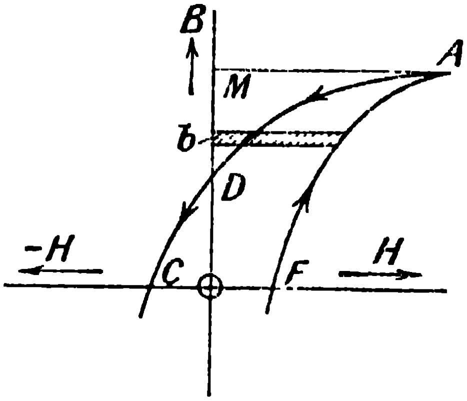


FIG. 77. ENERGY LOST IN HYSTERESIS

6. **Inductance in A.C. Circuits.** Inductances in circuits carrying alternating current produce effects which depend upon the values of the inductances and the frequency. As well as the e.m.f. of the source, there are the back e.m.f.'s of the

inductances, whether self or mutual. If $e$ is the e.m.f. of the source at any instant, $R$ the total resistance of the circuit, and $e_b$ the total back e.m.f. at that instant, then

$$e - e_b = Ri$$

by Ohm's law. It has been seen that $e_b = L\dot{i}$ and the expression can therefore be written

$$e = \left(\frac{L\dot{i}}{i} + R\right)i$$

In this form the effect of the inductance is taken into account by looking upon it as causing an addition $L\dot{i}/i$ to the resistance of the circuit. While the quantity $L\dot{i}/i$ does not depend upon the current strength in practical cases, it does depend upon the instant considered, and this arrangement of the equation is of no advantage in determining instantaneous values of the current throughout the cycle. But if, as is usually the case, only maximum or root mean square values are of interest, it is possible to combine the effects of the back e.m.f.'s with those of the resistance in a single quantity $Z$ known as the *impedance*, and to write

$$E = ZI$$

(*a*) INDUCTANCE ALONE. Consider first an ideal coil having an inductance $L$ henries and no resistance. Let it be connected to an a.c. source of p.d. $V$ volts and frequency $f = \omega/2\pi$. The vector diagram has already been drawn in Fig. 73. $OV$ represents the p.d. of the source and $OI$ the current.

$$V = \omega LI \text{ or } I = \frac{V}{\omega L}.$$

$\omega L$ is known as the *reactance* of the coil. In this case it is also the *impedance* of the coil, as it is the quotient of the p.d. divided by the current. As in the case of reactance in the condenser circuit (p. 58), the reactance of the inductance involves no expenditure of energy. While the field is being built up energy is taken from the source, but as the field collapses energy is given back. This arises quite clearly from the p.d. and current curves, which are drawn in Fig. 78 in the usual manner from Fig. 73. The power at any instant taken from the source is the product $vi$ and this is plotted as the dotted curve. From $O$ to $A$ the current and with it the field is increasing, and power is taken from the source. The total energy taken is the area of the $p$ curve $OP_1T_1$. From $A$ to $T_2$ the current and with it the field is decreasing, and power is given back to the source, as is shown by the curve $T_1P_2T_2$ being negative. The total energy given back is the area of the curve $T_1P_2T_2$ which is obviously equal to the area $OP_1T_1$. The energy given back is equal to the energy taken, and on the whole no power is taken from the source.

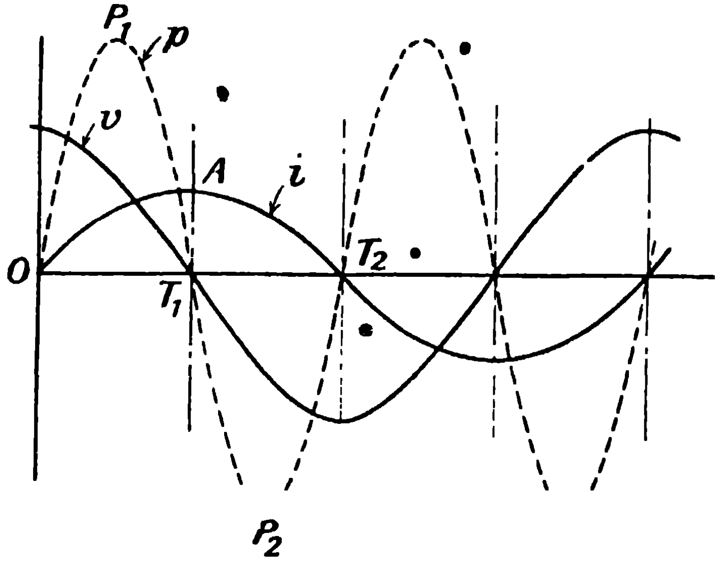


FIG. 78. CURRENT P.D. AND POWER CURVES

From the point of view of the vector diagram the state of affairs arises because the vectors representing the current and the p.d. are at right angles. Whenever this occurs, the power supplied to the circuit from the source is nil.

The reactance of an inductance should be compared with that of a capacitance (p. 58), and two important differences should be noted—

(i) The reactance of an inductance ($\omega L$) is directly proportional to the frequency, whereas the reactance of a capacitance ($1/\omega C$) is inversely proportional to the frequency.

(ii) In the case of an inductance the current curve lags behind the p.d. curve by a quarter cycle, i.e. the current vector is a right-angle behind the p.d. vector. In the case of a capacitance, the current curve is in front of the p.d. curve by a quarter cycle, i.e. the current vector is a right-angle in front of the p.d. vector. This difference arises because in the inductance case the p.d. is proportional to the rate of change of current ($v \propto \dot{i}$), but in the capacitance case the current is proportional to the rate of change of p.d. ($i \propto \dot{v}$).

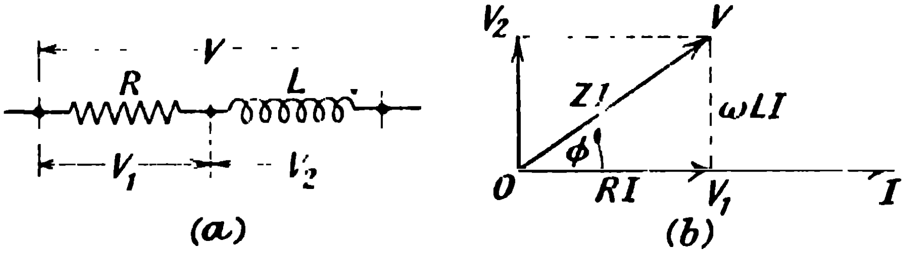


FIG. 79. RESISTANCE AND INDUCTANCE IN SERIES

These differences can be expressed in the vector diagrams by writing

$$V = j\omega LI$$

$$V = -\frac{j}{\omega C} I$$

in the two cases as an algebraic statement of the vector diagram, $j$ indicating a rotation of 90° forwards (counter clockwise) and $-j$ a rotation of 90° backwards (clockwise), and $V$ and $I$ being either maximum or root mean square values at will.

The statement then that the reactance of an inductance is $j\omega L$ and that of a capacitance is $-j/\omega C$, is complete; it includes both the points (i) and (ii).

(*b*) **INDUCTANCE AND RESISTANCE IN SERIES.** In the case of inductance and resistance in series, Fig. 79 (*a*), where the resistance $R$ includes that of the coil of inductance $L$, the p.d. $V$ of the source may be divided into two parts: one, $V_1$, across the resistance, and the other, $V_2$, across the inductance. If the current is $I$, $V_1 = RI$, and $V_2 = \omega LI$, and the vector diagram is drawn as in Fig. 79 (*b*). The current vector $OI$ is drawn horizontally for convenience $OV_1 = RI$ is the vector of the p.d. across the resistance; it is along the current vector. $OV_2 = \omega LI$ is the vector of the p.d. across the inductance;

it is at right-angles to $OI$ leading. The combined p.d. vector is $OV$, the geometrical sum of $OV_1$ and $OV_2$. Clearly

$$OV^2 = OV_1^2 + OV_2^2$$

i.e.

$$V^2 = R^2I^2 + \omega^2L^2I^2$$

$$V = \sqrt{(R^2 + \omega^2L^2)}I = ZI$$

where $Z = \sqrt{(R^2 + \omega^2L^2)}$,

$Z$ is called the *impedance* of the circuit. If $Z$ is known, and also $\phi$ the phase angle between $OV$ and $OI$, the current at any instant can be found from a given source by drawing the vector diagram and using the construction of Fig. 39.

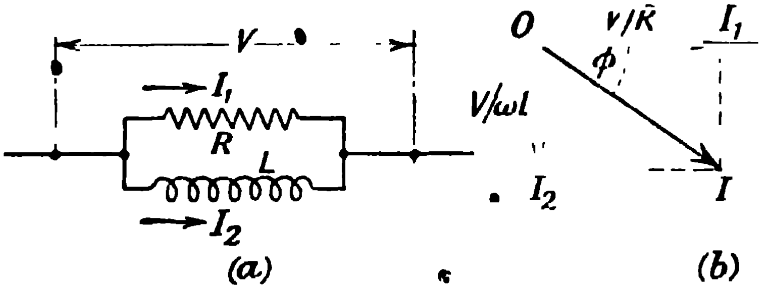


FIG. 80. RESISTANCE AND INDUCTANCE IN PARALLEL

Exactly as in the case of a capacitance, the power in the circuit is given by $\frac{1}{2}VI\cos\phi$ if $V$ and $I$ are maximum values, or by $VI\cos\phi$ if $V$ and $I$ are root mean square values.

Since the power required by the inductance is nil, and that required by the resistance is $RI^2$ ($I$ r.m.s.), then the total power is $RI \times I$, which is (projection of $OV$ on $OI$) $\times$ $OI$, which is $V\cos\phi \times I$, i.e. $VI\cos\phi$. Cos $\phi$ is the *power factor*.

(c) INDUCTANCE AND RESISTANCE IN PARALLEL. If a p.d. $V$ is established across a resistance $R$ and an inductance $L$ in parallel (Fig. 80 (*a*) ), the current $I_1$ through the resistance is $V/R$ in phase with the p.d., and the current $I_2$ in the inductance is $V/\omega L$ lagging 90° behind the p.d. The vector diagram is accordingly as drawn in Fig. 80 (*b*), where $OV = V$ is drawn horizontally for convenience, and $OI_1 = I_1$, and $OI_2 = I_2$. The total current is given by $OI$, and from the diagram,

$$I^2 = \frac{V^2}{R^2} + \frac{V^2}{\omega^2L^2}.$$

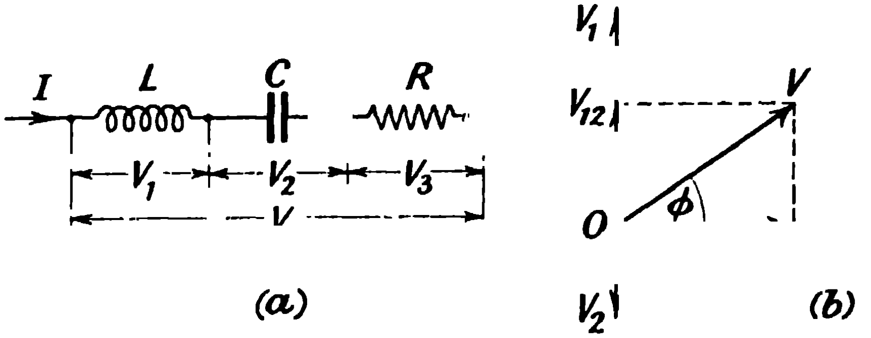
$$I = \sqrt{\left[\frac{1}{R^2} + \frac{1}{\omega^2L^2}\right]}\ V$$

These methods are readily extended to any circuit, however complicated.

(*d*) INDUCTANCE, CAPACITANCE, AND RESISTANCE IN SERIES. In this case, Fig. 81 (*a*), the current $I$ is the same in each, and the p.d. across the whole circuit is made up of three parts, $V_1$ across the inductance, $V_2$ across the capacitance, and $V_3$ across the resistance.

$$V_1 = \omega LI,\ V_2 = I/\omega C, \text{ and } V_3 = RI$$

and the vectors must be drawn as in Fig. 81 (*b*). The current vector $OI$ is taken horizontal for convenience, $OV_3 = RI$ is



(*a*) (*b*)

FIG. 81. RESISTANCE, INDUCTANCE AND CAPACITANCE IN SERIES

drawn along $OI$, $OV_1 = \omega LI$ is drawn leading $OI$ by 90° and $OV_2 = I/\omega C$ is drawn lagging behind $OI$ by 90°. $OV_1$, $OV_2$, and $OV_3$ are added geometrically and give the supply p.d. $OV$.

$$OV_{12} = OV_1 - OV_2 = \left(\omega L - \frac{1}{\omega C}\right) I$$

and

$$OV^2 = OV^2_3 + OV^2_{12}.$$

i.e.

$$V^2 = \left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right] I^2$$

$$V = \sqrt{\left[R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2\right]}\, I$$

The power is simply $RI^2$ ($I$ r.m.s.) as before.

The reactance of the circuit is the difference of the reactances of the inductance and the capacitance. A most interesting case occurs when these reactances are equal, for then the combined reactance is zero, and $V = RI$ as in a non-reactive circuit. This is known as the case of *resonance*. The energy required to establish the magnetic field of the inductance is taken from the electric field of the capacitance and *vice versa*; the source is not called upon at all except for the power required to heat the resistance. In this way $V_1$ and $V_2$ may be very large voltages, very much greater than the supply voltage $V$, if the reactances are much greater than the resistance.

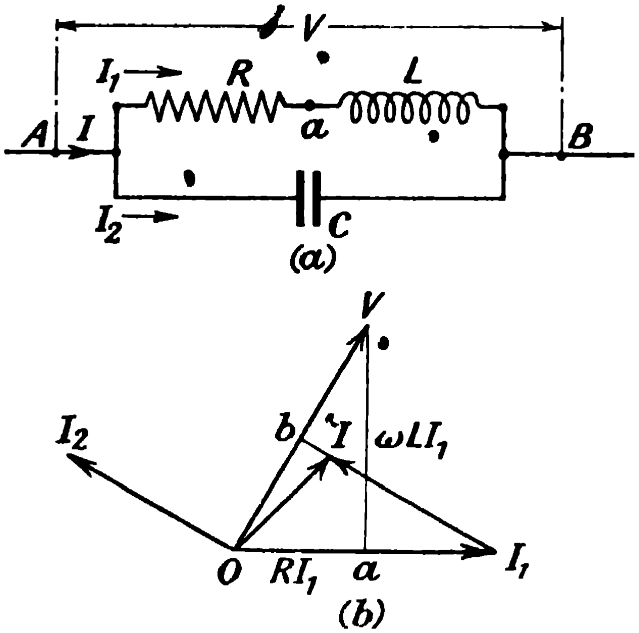


FIG. 82. CAPACITANCE IN PARALLEL WITH RESISTANCE AND INDUCTANCE

It should be noted that this resonance condition occurs when $\omega L = 1/\omega C$, i.e. when $\omega = 1/\sqrt{(LC)}$ or when the frequency is

$$f = \frac{1}{2\pi\sqrt{(LC)}}.$$

This series resonance is of great importance in wireless circuits.

(e) **INDUCTANCE AND RESISTANCE IN PARALLEL WITH CAPACITANCE.** Let, Fig. 82 (*a*), the current in the inductive branch consisting of $R$ and $L$, be $I_1$, and that in the capacitance be $I_2$. The total current from the supply $V$ is $I$. The vector diagrams for $I_1$ and $I_2$ can be drawn as above; the combined diagram is drawn in Fig. 82 (*b*).

$OI_1 = V/\sqrt{(R^2 + \omega^2L^2)}$, is drawn horizontally. Along $OI_1$ $Oa$ is marked $= RI_1$, and $aV = \omega LI_1$ is drawn vertically. $OV$ is the supply voltage. $OI_2 = \omega CV$ is drawn at right-angles to

$OV$ leading. $I$, the geometric sum of $OI_1$ and $OI_2$, is found as $OI$ by drawing $I_1I$ equal and parallel to $OI_2$.

$OI$ is in phase with $OV$ when $I$ falls on $OV$. This is the case of *parallel* or *current resonance*. To find the necessary condition, produce $I_1I$ to meet $OV$ in $b$. The resonance condition is that $I_1b = I_2 = \omega CV$. The triangles $OaV$ and $ObI_1$ are similar, therefore

$$\frac{bI_1}{OI_1} = \frac{aV}{OV}$$

$$bI_1 = \frac{OI_1}{OV} aV = \frac{I_1}{V}.\omega LI_1 = \frac{I_1^2}{V^2}.\omega LV$$

Now $bI_1 = \omega CV$ for resonance and $V^2/I_1^2 = R^2 + \omega^2L^2$

$$\therefore \qquad \omega CV = \frac{\omega L}{R^2 + \omega^2L^2}.V$$

or

$$C = \frac{L}{R^2 + \omega^2L^2}$$

If a capacitance of this value is shunted across any inductive apparatus such as a motor, the current and the voltage are brought into phase and the power factor is unity. Doing this is known as *power factor correction.*

The resonance condition may also be written in the form

$$\omega^2 = \frac{1}{LC}\left(1 - \frac{C}{L}R^2\right)$$

If $R$ is very small the condition is seen to be practically the same as for series resonance, i.e. $f = 1/2\pi\sqrt{(LC)}$. It should be noted that the currents $I_1$ and $I_2$ may in this case be very much greater than the total current $I$. Again, energy surges backwards and forwards between the magnetic and electric fields.

The impedance of the parallel circuit at resonance is very great if $R$ is very small. From the similar triangles $OaV$ and $ObI_1$

$$\frac{Ob}{OI_1} = \frac{Oa}{OV}$$

$$I = \frac{RI_1^2}{V} = R\left(\frac{I_1^2}{V^2}\right.$$

$$\frac{R}{R^2 \quad \omega^2L^2}V$$

and the impedance $Z$ is

$$Z = \frac{V}{I} = \frac{R^2 + \omega^2L^2}{R} = R + \frac{\omega^2L^2}{R}$$

which clearly becomes larger and larger as $R$ is made smaller and smaller. In the ideal case of $R = 0$, the impedance is infinity; no current is taken from the source, but currents equal to $\omega CV = V/\omega L$ flow in the parallel branches as energy surges backwards and forwards between them.

The large impedance of a parallel circuit is of great importance in wireless circuits.

(*f*) Circuits with Mutual as well as Self-inductance. In general mutual inductance can be dealt with in the same way as self-inductance, but care must be taken that the induced e.m.f.'s are reckoned in the correct direction. For instance, two coils wound on the same core may be connected in series in two different ways, in one of which the e.m.f.'s due to mutual induction are in the same direction as those due to self-induction, and in the other in the reverse direction.

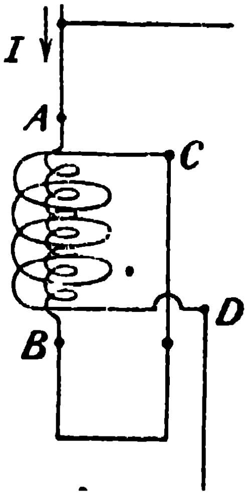


Fig. 83
Inductance and Mutual Inductance

Let the coils $AB$ and $CD$ (Fig. 83) be wound in the same direction. Then if $B$ is connected to $C$ and a current flows from $A$ to $D$ four linkages must be recognized, all in the same direction. They are (i) linkages due to the self-inductance of the coil $AB$ giving rise to an e.m.f. $L_1\dot{I}$, (ii) linkages with $AB$ due to the current in $CD$ giving rise to an e.m.f. $M\dot{I}$, (iii) linkages with $CD$ due to its self-induction giving e.m.f. $L_2\dot{I}$, and (iv) linkages with $CD$ due to current in $AB$ giving e.m.f. $M\dot{I}$. The total e.m.f. is the sum of these, i.e. $(L_1 + 2M + L_2)\dot{I}$, and $(L_1 + 2M + L_2)$ is the effective inductance of the two coils.

If, however, $B$ is connected to $D$ and the current flows from $A$ to $C$, the flux through $AB$ due to the current in $DC$ will be in the reverse direction to the flux through $AB$ due to the current through $AB$. Thus the induced e.m.f. due to the mutual inductance must be subtracted from that due to the self-inductance. The induced e.m.f. in the coil $AB$ is therefore $(L_1 - M)\dot{I}$ and similarly that in $DC$ is $(L_2 - M)\dot{I}$. The total e.m.f. in the two coils is $(L_1 - 2M + L_2)\dot{I}$, and the effective inductance of the combination is $(L_1 - 2M + L_2)$.

## CLASS EXPERIMENTS

### ON THE SUBJECT OF CHAPTER IV

1. With the apparatus of Fig. 53 carry out the experiments (i), (ii), (iii), (iv) and (v) described on page 117.

Plot the throw of the galvanometer on a base of—

(*a*) Number of turns in secondary;
(*b*) Area of secondary;
(*c*) Current in primary.

2. Wind solenoids on glass tubes of three different diameters putting on each the same number of turns per cm. and making the solenoids longer than, say, twenty diameters. Wind over the middle part of each solenoid a secondary coil of 100 turns with tappings to terminals at the 20th and 50th turns. This gives a choice of 20, 30, 50, 80 or 100 turns in the secondary coil.

(i) Connect a secondary coil to a ballistic galvanometer and find the throw when various currents are reversed in the primary coil. Plot the throw on a base of primary current.

(ii) Find the throw on reversal of a constant primary current with various secondary coils connected to the galvanometer.

Plot the throw on a base of number of secondary turns.

3. Calibrate a ballistic galvanometer to measure magnetic fields. Since the ballistic galvanometer measures quantity and, in the circuit of Fig. 53, quantity is proportional to change of linkages, a known field (that of the toroid) and a coil of a known area and number of turns are all that is required to carry out the calibration.

4. Calibrate the ballistic galvanometer, i.e. find the charge passed through it for a given throw by discharging a known condenser charged to a known potential, and thus find $k$ in the equation $Q = kd$. Since $I = E/R$ and $E = \dot{\Lambda} \times 10^{-8}$

$$Q = I = \frac{\dot{\Lambda}}{R} \times 10^{-8} \text{ or } Q = \frac{\Lambda}{R} \times 10^{-8} = kd$$

Calculate $\Lambda$ from the calibration and the results obtained in Experiment 2, and deduce the mutual inductance between the primary and secondary coils. Compare with the value of the mutual inductance calculated from the dimensions of the coils.

5. Use the calibrated ballistic galvanometer to measure—

(i) The flux linked with a bar magnet at different positions along its length.

(ii) The change of flux in an iron ring, wound uniformly with wire, when a measured current through this winding is reversed. Use gradually increasing values of magnetizing current, and draw a graph of flux density in the ring on a base of m.m.f. per cm. length. Reverse the current at each stage several times before taking the reading.

6. Measure an unknown inductance by comparing it with a known variable inductance.

Inductances can be compared by connecting them in a circuit like Wheatstone's bridge (see Fig. A). The arms $AB$ and $BC$ are ratio arms

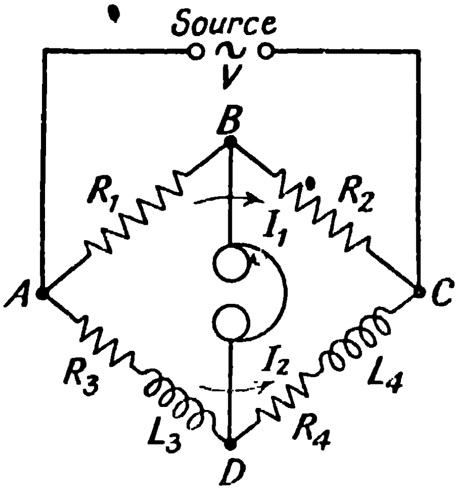


FIG. A

containing known resistances $R_1$ $R_2$ free from inductance. The two coils to be compared are $R_3$ $L_3$ in the arm $AD$ and $R_4L_4$ in the arm $DC$. In place of the battery a source of a.c. is used and instead of a galvanometer a telephone is connected across $BD$. If the frequency of the alternating current is not in the audible range an a.c. galvanometer is used. If no current flows in the telephones, $B$ and $D$ must always be at the same potential, that is, the p.d. $AB$ must always be equal to the p.d. $AD$.

Since the bridge must balance with any frequency, and since the values of the inductances do not depend upon the frequency, it must also balance with d.c., from which it follows (page 21) that one condition is

$$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

With a.c. impedances replace resistances, and it follows from the same considerations that

$$\frac{R_1}{R_2} = \frac{\sqrt{(R_3^2 + \omega^2 L_3^2)}}{\sqrt{(R_4^2 + \omega^2 L_4^2)}}.$$

which, since $R_1/R_2 = R_3/R_4$, gives

$$\frac{R_1}{R_2} = \frac{L_3}{L_4}$$

The double condition arises because it is necessary to balance the phase angle as well as the magnitude of the p.d.'s at $B$ and $D$. This is made clear in the vector diagram of the balanced bridge (Fig. B). Here $AC$ is the p.d. $V$ of the source. The vector of the current $I$ through the resistance branch will lie along $AC$, and that of the current $I_2$ through the inductance branch will lag behind $AC$ and be $AI_2$ say. Then $Ad$ along $AI_2 = R_3I_2$ is the p.d. across the resistance $R_3$, $dD = \omega L_3I_2$ drawn at right-angles leading $I_2$ is the p.d. across the inductance $L_3$, $Df = R_4I_2$ drawn parallel to $AI_2$ is the p.d. across the resistance $R_4$ and $fC = \omega L_4I_2$ drawn at right-angles to $Df$ is the p.d. across the inductance $L_4$.

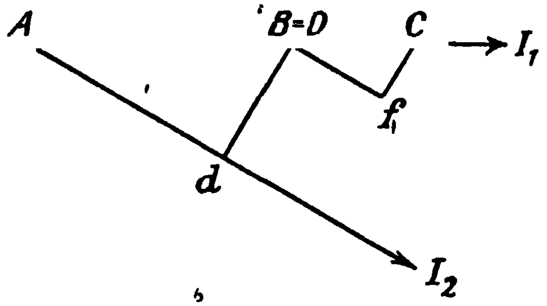


Fig. B

The p.d. $AB$ across $R_1 = R_1I_1$ and the p.d. $BC$ across $R_2 = R_2I_1$. Clearly for a balanced bridge $B$ must lie on $D$, and since $B$ necessarily lies in $AC$, $D$ must also lie in $AC$. It follows that the triangles $AdD$ and $DfC$ are similar and that

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} = \frac{L_3}{L_4}$$

This is not easy to arrange, if $L_3$ and $L_4$ are fixed. If one of them, say $L_3$, is variable, then by adjusting $L_3$ and also $R_3$ turn about, it is easy to balance the bridge. $R_3$ is adjusted by an extra adjustable resistance in series with the coil $R_3L_3$ which consists of two coils in series with one movable so as to vary the self-inductance of the combination. The value of the self-inductance at each position of the movable coil is read by a pointer on a scale. The variable self-inductance can be calibrated by using a standard inductance for $L_4$ and setting the ratio $R_1/R_2$ to different values and adjusting $R_3L_3$ for balance, then

$$L_3 = \frac{R_1}{R_2} L_4$$

Unknown inductances are then easily measured by the calibrated self-inductance.

### 7. Measurement of inductance and phase angle by the three voltmeter method

A coil possessing inductance has also resistance, and consequently there is no such thing as a pure self-inductance. When an alternating current is passing through an inductive coil the alternating p.d. between the ends of the coil depends on the impedance of the coil, and the p.d.

and the current are out of step by an angle $\phi$. In Fig. C (*a*) $AC$ = the p.d. on the coil (= $ZI$) is the hypotenuse of a right-angle triangle whose sides are $RI = AB$ and $\omega LI = BC$. The p.d. $AB = RI$ is in phase with the current $I$. The p.d. $AC = V_3 = ZI$ can be measured, but not $RI$ or $\omega LI$ separately. Now connect a resistance coil $R_1$, Fig. C (*b*), free from inductance in series with the coil $RL$ and measure the

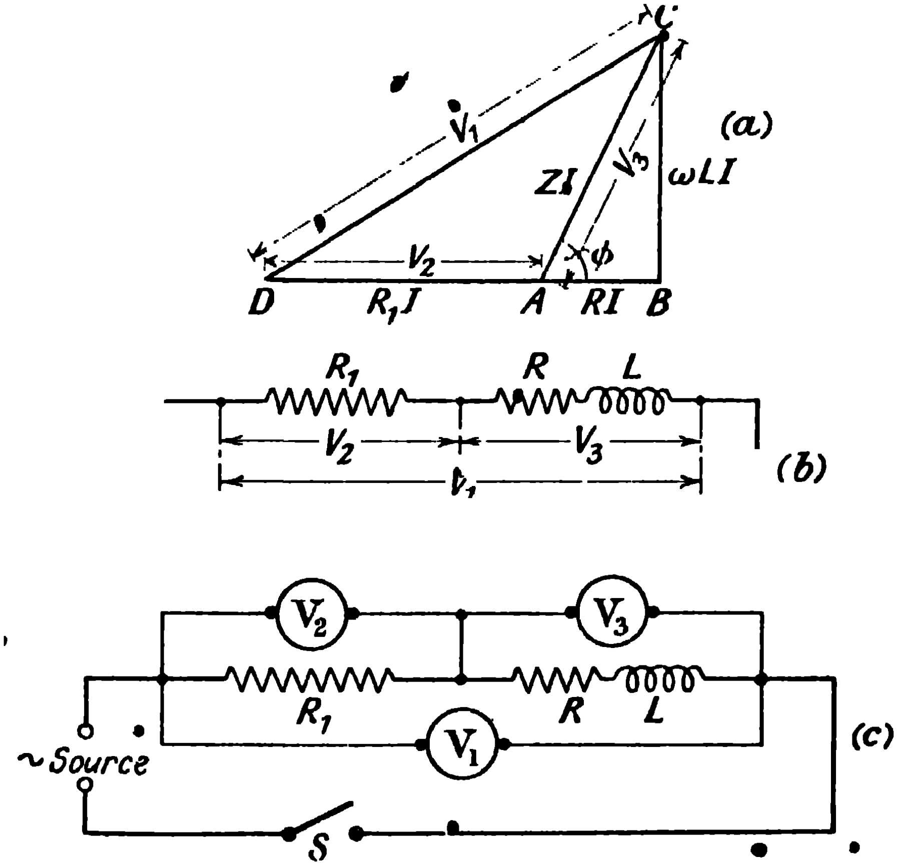


FIG. C

p.d. $V_2$ across it. $V_2 = R_1I$. Also measure the p.d. $V_1$ across the two coils in series. These different p.d.'s are connected with one another by the values of $R_1$, $R$ and $\omega L$ as in the diagram. The readings on the a.c. voltmeters, Fig. C (*c*), give the lengths of the lines $DA$, $DC$, and $AC$, so that this triangle can be drawn. Draw $CB$ perpendicular to $DA$ to meet $DA$ produced in $B$. Then $AB = RI$ and $CB = \omega LI$; the angle $CAB = \phi$.

$$R = \frac{AB}{DA} \times R_1 \qquad \omega L = \frac{CB}{DA} \times R_1$$

Hence $R$ and $\omega L$ are measured, and knowing $\omega$, $L$ is found. The measurement of the three p.d.'s may be made by connecting a single instrument in the three positions in succession.

8. Measure the inductance and phase angle of a coil of many turns with and without an iron core and with the iron circuit more or less complete. Do this at different frequencies, if possible, to find that the

reactance and therefore the phase angle of the coil increases with frequency.

9. Measure the inductances of two coils separately, $L_1$ and $L_2$. Connect them in series and measure the inductances of the combination $L_1 + L_2 + 2M$. Reverse the connections of one and measure again $L_1 + L_2 - 2M$. Hence find $M$ for various relative positions of the coils. Repeat the experiment with the coils on a common core of iron wire or stampings.

## EXERCISES

1. The p.d. at the terminals of a d.c. source is 149·1 V. when the source is supplying a current of 0·6 A. and 147 V. when the current is 2 A. Find the e.m.f. and internal resistance of the source.

(150 V., 1·5 Ω.)

2. The current through the 7 000 turns of an iron cored coil produces a flux of 500 000 lines. When the current is switched off the flux collapses in one-tenth of a second. What is the average value of the e.m.f. induced?

(350 V.)

3. An iron ring of mean diameter 30 cm. and cross-sectional area 5 cm.$^2$ is wound uniformly with 500 turns through which is passed a current of 2 A. A secondary winding of 250 turns is connected to a ballistic galvanometer; the combined resistance of the secondary winding and the galvanometer is 180 Ω. When the current is interrupted the galvanometer throw indicates a flow of 1 mC. Find the permeability of the iron.

(343)

4. A rectangular coil of 150 turns and lengths of sides 10 in. and 5 in. is rotating at 150 r.p.m. in a uniform field of 200 lines per cm.$^2$ in a direction at right-angles to the axis of rotation. Find the maximum value of the e.m.f. produced.

(10·1 mV.)

5. If the iron ring of Question 3 is replaced by a wooden ring, find the inductance of the toroid, and the mutual inductance between the two windings.

(525 μH.; 262 μH.)

6. An iron-cored coil has an inductance of 0·5 H. Find the energy that is stored in the magnetic field when a current of 2 A. flows in the coil.

What current must flow in order that 1 ft.-lb. of energy may be stored?

(1·0 J.; 2·25 A.)

7. If the flux of Question 2 is produced by a current of 5 A., what is the inductance of the coil on the assumption that all the flux threads all the turns?

(7 H.)

8. A hysteresis loop as drawn has an area of 0·5 sq. in. If the scales are 1 in. = 10 ampere-turns per cm. and 1 in. = 10 000 lines per cm.$^2$, what is the energy consumed in taking the iron through one cycle?

If 800 cm.$^3$ of the iron form the core of a coil and it is uniformly magnetized to the same maximum flux density as in the hysteresis loop test when the coil is connected to a 50 cycle a.c. supply, what power will be required on account of hysteresis?

(5 000 ergs; 20 W.)

9. A current of 2 A. 50 cycles flows through a coil having an inductance of 0·25 H. and a resistance of 30 Ω. What is the p.d. across the coil?

(168 V.)

10. A resistance of 20 Ω. is connected in series with the coil of Question 9, and the circuit thus formed is connected to a 220 V., 50 cycle supply. Find the current through the circuit, the p.d. across the coil, and the p.d. across the resistance. Check the answer by drawing a vector diagram.

(2·36 A.; 198 V., 57·2 V.)

11. Find the power absorbed in the coil of Question 9 and the power factor, if 2 A. is the r.m.s. value of the current.

(120 W.; 0·357)

12. A coil of inductance 0·1 H. and resistance 15 Ω. is connected in series with a condenser across a 220 V. 50 cycle supply. What must be the capacitance of the condenser in order that the resonance condition may be produced, and what then will be the current in the circuit and the p.d. across the condenser?

(99 $\mu$F.; 14·7 A.; 462 V.)

## CHAPTER V

# CHEMICAL ACTION AND ELECTRIC CURRENTS

1. **Primary Cells.** In the first chapter it was shown that, when electricity flows through some liquids, changes occur at the places where the current enters and leaves the liquid. The current must be led into and out of the liquid by metal plates. These are called *electrodes*, which means "ways for the electricity." The electrode by which the current enters the liquid is called the *anode* or the "way up stream," and the other by which the current leaves is called the *cathode* or the "way down stream." Liquids which are decomposed by a current are found to be better conductors than any other substances except metals. They are generally solutions of an acid or a salt in water. Pure water is almost an insulator, but it becomes conducting when some salt is dissolved in it. The conductivity is nearly proportional to the amount of salt dissolved in a given volume of solution. This shows that the current is carried by the dissolved substance. Another thing which confirms this is the important fact that the amount of the chemical action at the electrodes is exactly proportional to the quantity of electricity which has been passed through the solution.

A salt forming an electrolyte in solution is in general a compound of a metal and an acid radicle. When the current passes, it is found that the metal is deposited at the cathode and the acid radicle appears at the anode. The decomposition produced by the current is called *electrolysis*. If the electrolyte is copper sulphate and the two electrodes are platinum, copper is deposited as a coating on the cathode exactly in proportion to the amount of electricity which leaves the solution there. Clearly this means that each atom of copper carries the same amount of electricity out of the solution. The exactness of this makes it a convenient method of measuring quantities of electricity. Similarly, the amount of the acid radicle ($SO_4$) appearing at the anode is exactly proportional to the quantity of electricity entering the solution there. As the electrode in the case considered is platinum with which the acid radicle does not react, the radicle combines with water and decomposes, giving sulphuric acid and oxygen gas.* These are more difficult to collect and measure than the solid copper deposited on the cathode, and so are not used for measuring current. If the anode were a copper plate the metal could be attacked by the acid radicle arriving at it, and we should then have copper dissolved at the anode and deposited on the cathode.

The copper and the acid radicle move through the solution in opposite directions. They are driven by the electric forces which arise from the electric pressure or potential difference. So that in the same body of liquid there are equal quantities of each kind moving through one another. They carry opposite charges of electricity, so that they neutralize each other's effects, except where they arrive at an electrode at the end of their journey. The charges are called *positive* and *negative* to express their opposite characters. At the cathode a supply of carriers of positive charges arrives in a steady stream, and at the anode a supply of carriers of negative electricity arrives in a steady stream. This is the reason why it is only at the electrodes that the decomposition due to the electric current appears. The carriers are called *ions*. The positive ion is practically always an atom of a metal or an atom of hydrogen; occasionally, as in the Leclanché cell, it is a combination of atoms forming a radicle equivalent in chemical action to a metal atom. The negative ion is generally a combination of atoms forming an acid radicle. The actions taking place at the two electrodes are of opposite nature. Electrically they are expressed by stating the direction of the current, which is taken as the direction of movement of the ions called positive and is opposite to the direction of movement of the set of ions called negative. Chemically, metal is deposited at the cathode. The metal atom is freed from its combination with other atoms. This is the process called by the chemists *reduction*. At the cathode metal is dissolved from the electrode. This process is called *oxidation*. If the plate is not liable to attack there is a decomposition set going by the ion on arrival, which liberates oxygen.

The chemical decompositions produced by the passage of a current through an electrolyte have many applications in manufacturing processes. In fact, some of the products obtained in this way cannot be got in any other way, except at great expense. For instance, aluminium is separated by the electrolysis of a bath of fused cryolite. Before this method was developed, the metal was so expensive that it was rather a curiosity. Soda (and chlorine) are obtained from the electrolysis of brine much more cheaply than by the older process based on the action of sulphuric acid on sea salt followed by further complicated treatment of the product. The refining of copper for electrical purposes is now carried out exclusively by means of electrolysis. The anode is a plate of the crude copper as delivered from the furnaces in which it has been reduced from its ore. The electrolyte is a bath of copper sulphate solution and the cathode a sheet of pure copper. A fairly high current density is used (20–30 A./sq. ft.) and the temperature of the bath is controlled. The copper deposited on the cathode is free from the impurities present in the original ingot, as the conditions of current density, etc., do not allow them to take part in the electrolysis. They collect in a kind of mud on the anode. As the resistance of a copper conductor may be increased enormously by a very small percentage of impurity—too small to be removed by other refining methods—wire of a reliably high standard of conductivity was not available until electrolytic refining was adopted.

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* $H_2O + SO_4 = H_2SO_4 + O$

Electrolytic meters have been designed which measure the total amount of electricity delivered in any time by the amount of mercury reduced from a solution. The anode is a pool of mercury at one end of a tube and the amount collected at the cathode measures the consumption. By tipping up the tube the mercury flows back to the anode and the meter is reset.

Another example of electrolysis is worthy of study. When a current is passed through a dilute acid solution with platinum electrodes, hydrogen ions arriving at the cathode and oxygen ions arriving at the anode are liberated as bubbles of gas. If these are collected in tubes placed over the electrodes for that purpose the gases may be stored. They are evolved in the proportions of two volumes to one. These are just the proportions required for complete combustion of the hydrogen. At the same time the corresponding amount of water has disappeared from the solution. Now if the hydrogen is burnt with the oxygen there is a very hot flame; a large amount of heat is produced. Heat, we know, is a form of energy, so that in the combination of these two gases to reform the water from which they were produced, we obtain a large supply of energy. Can an energy profit be made by this means? Can we get energy continually out of this treatment of water without putting an equal amount in? Clearly from the law of conservation of energy it is impossible. At least as much energy must be put into the electrolysis of the water as is got out in the form of heat in the burning of the products. This energy must be put in to drive the current through the cell in which the electrolysis is performed. There is on this account an opposition to the passage of the current in addition to the friction in any resistance in the cell.

Let the current be $I$ amperes and the potential difference at the input $V$ volts, then

$$VI = \text{watts} = \text{joules supplied per sec.}$$

and if $R$ is the resistance concerned

$$I^2R = \text{watts spent as heat in the resistance.}$$

$$VI - I^2R = \text{watts spent in the decomposition.}$$

We can calculate this quantity, as we can measure the number of calories produced by burning a measured amount of hydrogen and oxygen. Each ampere produces the same weight, $W$ grammes of hydrogen per second. Burning 1 g. of hydrogen gives us say $q$ calories of heat. Consequently the energy put in to decompose the water is

$$WqI \text{ cal. per sec.} = WqI \times 4{\cdot}2 \text{ joules per sec.}$$
$$= 4{\cdot}2WqI \text{ watts}$$

Thus
$$VI - I^2R = 4{\cdot}2WqI$$
$$V - IR = 4{\cdot}2Wq$$

We see that Ohm's law does not apply to an electrolytic cell unless we take account of this quantity $4{\cdot}2Wq$.

If we write this statement in a different order

$$V - 4{\cdot}2Wq = IR$$

we see that Ohm's law $V = IR$ holds if a back electromotive-force $4{\cdot}2Wq$ is reckoned in as being subtracted from the input potential difference measured at the terminals of the cell. In order to get a current through the cell $V$ must be greater than $4{\cdot}2Wq$ volts. This result is easily verified practically.

Connect up a circuit with an electrolytic cell, ammeter, voltmeter and a rheostat with a battery of, say, 4 volts.

Plot readings of the current against the voltmeter readings and a result like that in Fig. 85 is obtained. The current is

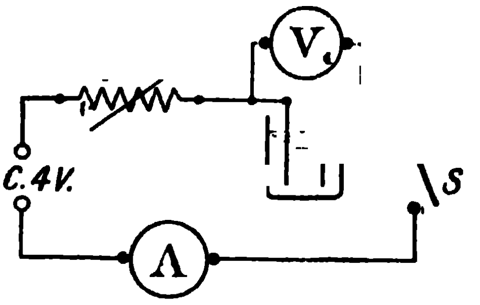


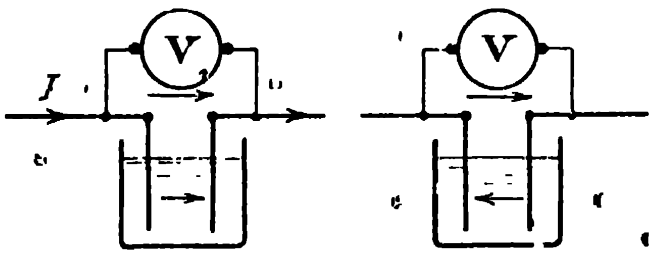
FIG. 84. ELECTROLYSIS

FIG. 85. BACK E.M.F.

proportional to the excess of $V$ over a quantity $e$. The reciprocal of the slope of the line is a measure of the resistance $R$ of the cell, for

$$R = \frac{V - e}{I} = \frac{BC}{AB} = \frac{1}{\text{Slope}}$$

If the switch $S$ is opened the voltmeter continues to give a reading which begins at the value $e$ and slowly falls. The voltmeter indicates really the current which is passing through it, and, consequently there must be a potential difference supplied by the cell to drive this current. Consider the currents through the cell and voltmeter before and after the switch $S$ is opened. Their directions are indicated in Fig. 86.



*Before opening switch.* *After opening switch.*

FIG. 86. ACCUMULATOR

The current through the voltmeter is in the same direction as before. This is shown by the direction of the deflection of the pointer being the same. We know, therefore, that the current through the cell has reversed. If we examine the electrodes in the cell, we shall see that when the switch is opened there are bubbles of gas on both of them. As the cell continues to supply current this gas gradually diminishes and disappears.

These gases are hydrogen on the cathode and oxygen on the anode. On opening the switch, the current in the cell reverses and the old anode is now the cathode, and the old cathode becomes the anode. This means that hydrogen is being delivered at the electrode covered with oxygen and oxygen at the electrode with a layer of hydrogen on it. There is a quiet re-combination of hydrogen and oxygen going on at the two electrodes, and the energy of their re-combination is being spent in sending the current shown by the voltmeter. This shows that not only can the energy of the electric current be used to produce chemical changes, but, conversely, the energy of chemical changes can be used to supply electrical energy.

The first method of producing a steady flow of electricity to be discovered was by means of two metal plates in a solution of an acid or salt. When the two plates were of different metals, a current was found to flow through a wire joining the plates. The wire got warm. To drive the current through the resistance of the wire needed a supply of energy. We can now see where this energy comes from. It comes from the energy evolved in the chemical reactions which go on in the cell. If the plates of such a cell are examined, one will be found to be eaten away and dissolved. If a piece of this metal is taken and dissolved in the acid of the cell, heat will be found to be evolved, i.e. there is a setting free of energy. If the current in the cell is reversed by a supply of electricity of higher potential difference, the action is reversed and the metal is deposited on the electrode from the solution. But not only the metal is put back. The energy set free when the metal dissolved has also to be restored to it. There is therefore an opposition to the current flowing in this direction like the opposing voltage *e* in the last experiment, because work must be done to move the electricity in this direction.

Here we have a way in which energy can be supplied to an electric circuit to maintain a current in it. If the cell is not connected to a complete circuit so that a current can flow, the chemical actions go on for a time and then stop. The energy is spent pushing positive electricity into one plate and negative electricity into the other until the congestion in the plates becomes so great that the work required to push any more into either is equal to the energy that is available from the chemical actions. A deadlock is reached with a certain potential difference between the plates—the potential difference on open circuit. This potential difference measures then the energy liberated in the cell for each coulomb of electricity it delivers. It is a definite amount for each make of cell and is called the *electromotive force* of the cell. *The electromotive force of any electric generator is defined as the energy supplied to each unit of electricity sent out.* It is measured practically in joules per coulomb, or watts per ampere, or volts, and is generally represented by the letter $E$ or $e$.

When a cell is supplying a current $I$, the energy supplied is $EI$ joules per second. This is spent in the circuit in various ways. One way, which must not be overlooked, is in sending the current through the resistance of the cell itself. The electrolyte in the cell has a resistance just as the metal conductors used to make up the external circuit have a resistance. If this internal resistance is $r$ ohms, the energy spent per second is $I^2r$ and the power output from the cell is reduced by this amount. If the potential difference at the terminals of the cell is $V$, then we have the power output $VI$, so that

$$VI = EI - I^2r$$

$$V = E - Ir$$

or

$$E - V = Ir$$

Internal resistance in a cell is clearly to be avoided as far as possible, as it results in a pure loss. Considering the electrolyte in the cell as a conductor, its resistance is diminished by making the path of the current through it as short as possible and by having the plates as large in area as possible. The resistance depends on the size and nearness of the plates; on the other hand, the electromotive force $E$ in no way depends on these things but merely on the materials of which the cell is made up.

When a metal plate is put into an electrolyte, chemical action takes place, and in consequence of this the plate gets an electric charge, and we get a potential difference between the plate and the solution. The action stops when the plate is charged up to a potential difference equal to the electromotive force developed in the chemical action. If the potential difference were made greater by any means, the actions would reverse, undoing what had been done and lowering the potential difference between the plate and the solution to its original value. When we have two metal plates we have a potential difference between each and the solution, $e_1$ and $e_2$. The p.d. shown at the terminals is

$$E = e_1 - e_2$$

If the two plates are of the same metal $e_1 = e_2$ and $E = 0$. The metals must be different if we are to have a useful cell. $E$ also depends on the solution used in the cell, as changing it changes both $e_1$ and $e_2$. Many different types of cell have been devised, as there is a large choice of materials available, both for the plates and the solution. However, in practice the variety is not great. Our choice is guided by considering efficiency and cost. Starting with the original simple cell, as invented by Volta, having two metal plates in a dilute solution of an acid, it is not a difficult matter to find what pair of metals gives the highest potential difference. They can be compared by means of a high resistance galvanometer, or a voltmeter. The different pairs are then considered from the point of view of cost.

The best pair will be found to have zinc as one member, and carbon, copper, gold or platinum as the other, with no great difference between them. The matter of cost can be settled at once in these circumstances. All cells have now zinc paired with carbon or copper as electrodes.

We will now examine the cell more in detail. Making up a cell from plates of zinc and copper we find no serious difference in the current sent round a given circuit using different acid solutions, that is, at the beginning. But if we leave the cell connected, the current gradually dies down and finally stops. The zinc plate looks all right, cleaner if anything than before. The copper plate is covered with bubbles of gas. If these are wiped off, the current is partly restored, but not for long. Only if the plate is taken out and thoroughly scraped does the current recover its original value. The passage of the current has changed the nature of the surface of the plate. This plate is the cathode of the cell and the gas which collects there is hydrogen, as in the case of the electrolysis of a dilute acid which has been discussed. The film of hydrogen on the plate has changed the nature of the plate in contact with the electrolyte, and consequently the part of the electromotive force due to the contact. In the equation $E = e_1 - e_2$, $e_2$ due to the copper plate is changed to the value for a hydrogen plate, which is higher, and so the difference $e_1 - e_2$ is made smaller until it vanishes. This effect is called *polarization*. The principal effect of polarization is this opposing electromotive force or back e.m.f. There is another effect which also reduces the current given by the cell. This is, that the resistance of the film on the plate to the passage of electricity is high, and consequently the current is diminished. These two results of the polarization make the simple cell unsuitable for most purposes, and several methods have been devised to get rid of it. These are—

(*a*) The provision of a substance in the solution which will combine with the hydrogen as fast as it collects on the plate.

(*b*) Putting something on the plate which will combine with the hydrogen, and thus prevent the formation of the film.

(*c*) Arranging the cell constituents so that hydrogen is not deposited on the plate, and consequently no provision is required for its removal.

We will consider present-day cells in which each of these three methods is employed.

*Method* (*a*). Of the cells which make use of this method, the only type likely to be met with is the Bichromate cell. This cell has electrodes of zinc (Zn) and carbon (C) and the electrolyte is a solution of sulphuric acid ($H_2SO_4$) with the addition of bichromate of potash ($K_2Cr_2O_7$) or of chromic acid ($H_2CrO_4$). These two substances are compounds containing a combination of chromium with a great deal of oxygen, in fact $CrO_3$. The hydrogen liberated at the carbon plate combines with some of this oxygen and reduces it to the lower oxide $Cr_2O_3$. The electrolyte when fresh has a deep orange colour, which changes to dark green as the chromic acid is converted to chromium sulphate ($Cr_2(SO_4)_3$). The electrolyte needs to be renewed when the solution reaches this colour. As the electrolyte will dissolve zinc even when no current is passing, the zinc plate must be lifted out when the cell is not in use. Owing to this complication, this cell is rarely used nowadays. Its virtues are low resistance and high electromotive force, 1·7 volt. It is therefore capable of a high current output. The spread of the supply of electric power and the development of the accumulator have made it practically obsolete.

*Method* (*b*). Cells depolarized by this means, on the other hand, have not yet been displaced from general use, as they have special advantages of portability and reliability while requiring the minimum of attention. The typical cell is the Leclanché cell. The electrodes are zinc and carbon and the electrolyte is a solution of sal ammoniac. The depolarizing agent is black oxide of manganese ($MnO_2$). This metal manganese ($Mn$) is similar to chromium ($Cr$) in its chemical properties and also forms several compounds with oxygen. One of the richer oxides is the black oxide. This is cheap and occurs naturally as one of the ores of manganese called pyrolusite. This is powdered and mixed with carbon to make it sufficiently conducting. The mixture is either packed tightly round the carbon plate in a porous pot, or is compressed into blocks which are held to the carbon plate by rubber bands. Sal ammoniac ($NH_4Cl$) is a compound of ammonia ($NH_3$) with hydrochloric acid ($HCl$). Zinc is dissolved when a current is being generated and the compound ion $NH_4$ travels to the carbon plate. In contact with the mixture at this electrode, the excess hydrogen forms water, reducing the black oxide to one with less oxygen, and the ammonia is allowed to escape. A gas vent must therefore be provided if the mixture is packed in a pot. The solution of sal ammoniac is not so vigorous in acting on the zinc as an acid, so that the cell can be left practically indefinitely with no other attention than replacement of the water lost through evaporation. The oxide of manganese being insoluble, the oxidation of the hydrogen liberated at the electrode is not so rapid as if it were in solution, and so the depolarization may not keep pace with the requirements of the cell if the current is large. The e.m.f. of the cell, therefore, will fall gradually when it is supplying a current. The resistance may also be increased because of the pores of the mixture at the carbon electrode becoming choked with bubbles of gas. However, when the circuit is opened, the accumulation of hydrogen ceases and the depolarizer gradually overtakes the arrears, the electromotive force rises to its normal value, and the resistance is restored to its original value. The normal e.m.f. of this type of cell is about 1·5 volts, and the resistance of the quart size of cell is about ½ ohm. Where the supply of electric current is only required intermittently with periods of rest, and a high voltage is not wanted, as for house bells, alarms, etc., this type of cell is particularly useful. The electrolyte is not a dangerous corrosive liquid, and the cell is always ready for immediate use without any preliminary attention. A variety of this cell is the *dry* cell in which the electrolyte is made pasty with some absorbent porous material so that it cannot spill. A large number of small size dry cells connected in series provides a battery which can give a high potential difference, if only a small or moderate current is required. This is the pattern used for "high tension" batteries for wireless reception, and in various sizes for portable electric lamps. The electromotive force of this type of cell falls somewhat during life owing to the using up of the electrolyte, etc., and the resistance increases. If the cell is called on to supply a current for a long time continuously the electromotive force gradually falls owing to polarization. This fall is the more rapid the heavier the current. The original electromotive force is largely recovered when the cell is left idle. Many patterns are produced by different makers using different materials to render the electrolyte non-spillable, and also using other salts instead of, or in addition to, sal ammoniac.

*Method* (*c*). The outstanding pattern of cell which avoids polarization altogether is the Daniell cell. The electrodes are zinc and copper. The electrolyte is separated into two parts by a porous pot of unglazed earthenware. The fluid round the zinc plate or rod consists of dilute sulphuric acid ($H_2SO_4$) or a solution of zinc sulphate ($ZnSO_4$). The copper plate is immersed in a concentrated solution of copper sulphate ($CuSO_4$). When a current passes through the cell from the zinc to the copper electrode, zinc is dissolved from the anode and copper is deposited on the cathode. Consequently the electrodes are unchanged by the passage of the current. Polarization does not occur. If a current is sent in the other direction, copper is dissolved from the copper anode, and zinc is deposited on the zinc cathode. The cell is thus reversible in principle. As the energy set free by the solution of zinc in the cell gives an electromotive force of about 1·8 volt = $e_1$; and the energy from the solution of copper gives an electromotive force of about 0·8 volt = $e_2$, the resultant e.m.f. is in the direction which brings zinc into solution and deposits copper. The net e.m.f. $E = e_1 - e_2 = 1{\cdot}0$ volt approximately. As polarization does not occur, this e.m.f. is constant and the cell can be used as an approximate standard of potential difference. The resistance of the ordinary pattern of Daniell cell is about ½ ohm when in good order. The two solutions tend to mix through the pores of the earthenware pot, but while the cell is delivering a current the zinc ions are travelling through the pores of the pot and the copper ions are travelling in the same direction towards the copper plate. The presence of zinc in solution in the copper compartment does not affect the working of the cell, but when the cell is idle, copper ions may diffuse into the zinc compartment. On meeting the zinc electrode a copper ion exchanges with a zinc ion, which goes into solution and copper is deposited on the zinc. Then we have a little local cell complete and short-circuited where the copper touches the zinc. This will give rise to local currents circulating actively wherever a speck of copper is deposited and the zinc is eaten away uselessly. Further, on putting the cell into service again, little or no energy is available in the external circuit, as the energy got from the solution of the zinc is frittered away in this "local action." To prevent this, the cell should be dismantled and the solutions separated when it is not being used, or, as an alternative, the cell may be connected to a circuit of moderately high resistance, so that it supplies a small current; this causes a drift of the copper ions towards the copper plate and prevents them from diffusing into the zinc compartment. The Daniell cell is often called a *closed circuit cell* because it may not be left idle on open circuit. (The Leclanché cell is, for similar reasons, termed an *open circuit cell.*) As copper is continually being deposited on the copper electrode from the solution, provision must be made to keep up the supply by having crystals of copper sulphate on a shelf in the cell in contact with the liquid.

It has been pointed out above that the Daniell cell, being free from polarization, might serve as a standard of potential difference. However, as its electromotive force is liable to fall if the zinc compartment becomes contaminated with copper sulphate, it is not a suitable cell to keep ready made up for such a purpose. Latimer Clark designed the cell named after himself, in which the copper of the Daniell cell is replaced by mercury. A layer of mercurous sulphate ($Hg_2SO_4$) is placed over the liquid mercury; this, being slightly soluble in water, makes this electrode reversible as is the copper electrode in the Daniell cell. The other electrode is a rod of pure zinc, and the electrolyte is a concentrated solution of zinc sulphate. There is no porous pot or any partition in the cell to divide it into two compartments, as the presence of mercury on the zinc rod produces no ill effects. When carefully prepared with pure materials the e.m.f. of the cell is 1·4328 volts at 15° C. This cell formed the legal standard of potential difference at one time. Though the mercury electrode is reversible, i.e. mercury is dissolved or deposited at the electrode according to the direction of the current, it is found that, if more than a minute current is taken from the cell, the mercurous sulphate in solution near the electrode is all used up and, owing to the low solubility of this substance, it is only replaced very slowly by solution of the solid present. Thus the cell is liable to polarization unless care is taken, and then the potential difference at the terminals has not the correct value. This cell has also a temperature coefficient, largely due to the variation in the strength of the electrolyte as the temperature changes. The Latimer Clark cell is now replaced by the Cadmium cell. This is a similar cell merely replacing the zinc of the Latimer Clark cell by an amalgam of cadmium and the solution of zinc sulphate by one of cadmium sulphate ($CdSO_4$). It is not so sensitive to variations of temperature and recovers more rapidly from the effects of supplying more than a very small current. It has been adopted by all countries as the legal definition of the Standard Volt. When made up to the standard specification, the potential difference between the terminals is defined by Act of Parliament to be 1·0183 volt at 15° C. This fixes, as nearly as it is possible to do so at present, a standard unit of potential difference which would deliver 1 watt of power in maintaining a current of 1 ampere.

The Weston cell is similar to the standard cadmium cell, but is made up to a slightly different specification for the strength of the electrolyte, etc. It has a slightly different e.m.f.

These cells are never used to supply a current, but to provide a standard of potential difference for the calibration of voltmeters and similar purposes by the potentiometer method.

The Daniell and Leclanché cells on the other hand are used for the supply of current, and their electromotive forces are liable to some variations from changes in temperature, strength and purity of the solution forming the electrolyte. The internal resistances of cells are liable to wide fluctuations, particularly those patterns which use a porous pot as the pores may get more or less choked in the course of time.

2. **Secondary Cells.** The energy which drives the current, when supplied by the cells described in the last chapter, is obtained from the chemical energy of combination of zinc. This is a very expensive fuel, but it possesses the advantage that practically all the energy of the combustion of the metal is converted into electric energy with little or no waste. But even this efficiency does not make electric energy cheap when obtained in this way. With zinc at £14 a ton, the cost is 1½d. a pound. As 0·000339 grammes of zinc are dissolved for each coulomb supplied and the e.m.f. of a Daniell cell is 1·1 volt, approximately $1{\cdot}47 \times 10^6$ joules can be obtained in the form of electric energy from the combustion of 1 pound of zinc in Daniell cells. This is a little over ⅖ kWh. The expense for zinc alone would be about 4d. per kWh. if drawn from primary cells. In addition, the costs of manufacture, installation, renewals, and maintenance must be taken into account. The cost of energy obtained in this way is high. In general, the price of the raw materials is the least part of the cost of any article. This is specially true in this case. For example, the cost of a flash lamp battery is sixpence. It can supply 0·25 amperes for about eight hours at an e.m.f. which is 4·5 volts at the beginning and falls to 3 volts at the end of its life. This gives an energy output of 7·5 watt-hours, approximately, say 1¼ watt-hours for 1d., or for 1 kilowatt-hour, 800d., = £3 6s. 8d.

The great convenience and simplicity of control of electrically operated devices as well as the possibility of producing results by means of the electric current which could only be obtained with great difficulty by other means, if at all, has given electric cells a position of usefulness where they have no rival in spite of their cost.

The cost of a pound of coal is much less than that of a pound of zinc, and it will yield a very much larger supply of energy when burnt. But this energy is in the form of heat.

A certain proportion is lost up the chimney, but the greater part of it can be converted into the energy of high pressure steam in a boiler. This again can be converted in a steam engine into mechanical energy which can be converted into the energy of an electric current in a dynamo or alternator. The efficiency of the conversion of heat energy into mechanical energy is low, for only about 20 per cent is converted in the steam engine. On the other hand, the efficiency of a generator is high and practically all the mechanical energy input is available in the electrical output. As a result of the cheapness of the original store of chemical energy in coal and air, and in spite of losses in changing from one form of energy to another, the cost of electrical energy in bulk, generated from coal, can be kept under 1d. per kWh. in towns. This includes all costs of machinery, buildings, rent, rates, fuel, staff, and distributing mains.*

The cheapness of electric energy generated in this way depends in part on its being used as it is produced. If it is not being used much of the plant is still running and the cost of idle machinery must be allowed for. A demand for energy, which rises and falls at different times and seasons, makes it expensive to the supplier, as the capacity of the plant must be greater than the peak demand. It was felt to be desirable to discover some way of storing the energy at slack times so that it might be available to assist in supplying the demand during busy hours. A French scientist, Planté, studied how this storage of energy might be arranged for by electrochemical means. He examined the suitability of different metals and electrolytes for the purpose, and found that an electrolytic cell using lead plates as electrodes and dilute sulphuric acid as electrolyte gave the most satisfactory results. It will make things clearer if we examine what the problem was. We know from what has been said about the passage of electricity through electrolytes, that on reversal of the direction of flow of the electricity the chemical actions at the electrodes are reversed. Or rather, they would be reversed if the materials, produced at the electrodes when the flow was in one direction, remained there to take part in the changes when the direction of flow was reversed. Thus, when sending a current through dilute sulphuric acid, hydrogen is set free at the cathode and oxygen at the anode, these being obtained from the decomposition of the water. At the same time energy must be supplied to cause this decomposition. When the current passes in the reverse direction through the cell as in the experiment on p. 156, the anode and cathode exchange with one another and the action at each is reversed. Energy is set free by these actions to drive the current. Hydrogen set free at what is now the cathode combines with the oxygen clinging there and reforms water, giving out the energy of combination as electric energy, and not as heat. A similar action occurs at the other electrode. The e.m.f. generated depends on the energy liberated per coulomb. The number of coulombs which can be delivered by this reverse action depends on the amount of the products of the direct current which are stored and available. It is this latter factor which decides the capacity of the cell as a storer of energy—the number of ampere-hours it will supply when charged—for the e.m.f. is fixed for any given type. Planté found that with lead plates in dilute sulphuric acid the e.m.f. which was available was about 2 volts, and that the amount of energy which could be stored was large compared with most other possible materials.

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* In isolated and remote country districts the expense of distribution may raise the cost to 10d. per kWh.

These results are a consequence of the properties of the metal lead. The products of the electrolysis when energy is being put into the cell must be stored at the electrodes and not allowed to stray. They must therefore be insoluble and not liable to pass away in gas. Also they must be reasonably good conductors of electricity, for, if not, then the current passes them by and they take no further part in the cell. The action at the anode is a liberation of oxygen which must be stored. The electrode is to be oxidized. At the cathode a supply of hydrogen is set free. This will not combine with the metal, but if the metal is oxidized it will take the oxygen away, forming water, and thus reduce the oxide to one with less oxygen or even to the pure metal. This may be set down simply,

At anode . . . oxidation
At cathode . . . reduction

(*a*) The Lead-acid Cell. Lead is one of the metals which forms many compounds with oxygen. Several of them are well known and they have a wide range of uses.

Expressed in symbols the ones which are of interest in this case are—

| | | |
|---|---|---|
| $Pb_2O$ . . . | Lead suboxide | Grey |
| $PbO$ . . . | Lead oxide, litharge | Pale Yellow |
| $PbO_2$ . . . | Lead peroxide | Light brown |
| $Pb_2O_5$ . . . | Lead pentoxide | Black |

In addition there are other oxides which do not occur in the cell. One that is well known is red lead, with the formula $Pb_3O_4$, and is obtained by heating lead in air. When a lead plate is used as an anode in dilute sulphuric acid, it is gradually oxidized. The stages are Pb, $Pb_2O$, PbO, $PbO_2$, $Pb_2O_5$. The formation of the first two oxides is found to give out heat, or in the cell, energy, which assists the passage of the current. The latter pair of oxides, however, require a supply of energy to enable them to form. They set this energy free when they are decomposed or reduced. If the surface of the plate is already oxidized and it is used as a cathode, the changes take place in the reverse order, from $Pb_2O_5$ to $PbO_2$, then PbO, $Pb_2O$ and Pb in succession. The first two stages in the reduction liberate energy to assist the current and the latter two absorb energy. In fact, lead and oxygen combine with evolution of energy up to a certain stage—PbO—but to get a further quantity of oxygen to combine to form one of the higher oxides needs the input of energy. These higher oxides are not so stable and are easily induced to part with their surplus oxygen and thus revert to the very stable litharge PbO.

When the anode has been oxidized to $Pb_2O_5$, any further current causes no further oxidation to take place, the oxygen liberated passes off in bubbles, and similarly at the cathode, when all the oxide has been reduced, any further liberation of hydrogen causes bubbles of gas to come off. This is what is meant by saying the plates are "gassing."

There is no further storage of energy after this stage. If wires connected to the two plates are now connected, a current will flow in such a direction in the cell that energy is drawn from the chemical actions to drive the current round the circuit. This will result in the highly-oxidized plate being reduced and in the reduced plate becoming oxidized. When both plates are brought to the stage of PbO there is no further supply of energy and the current ceases. There is no e.m.f. in the cell. On passing a current from another generator through the cell it can be recharged and is then ready to repeat its output. The e.m.f. of the cell when charged is about 2 volts. This varies slightly with the strength of the acid and the temperature. The number of ampere-hours which the cell can deliver when charged depends on the amount of active oxide on the surface or in cavities in the plates. The plate which acts as anode during charge and by which the current leaves the cell during discharge is called the *positive* plate. The other is called the *negative* plate.

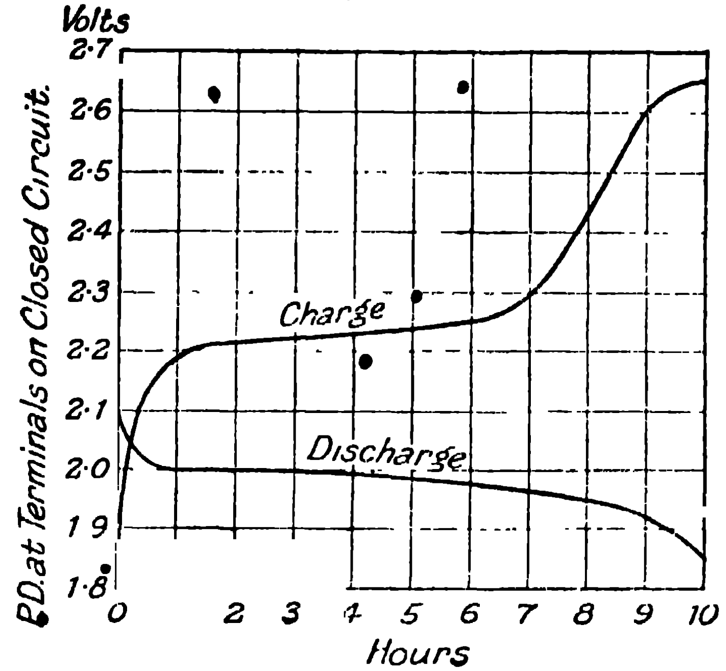

FIG. 87. CHARGE AND DISCHARGE CURVES OF LEAD ACCUMULATOR

The changes taking place during these operations can be set out in the following manner —

| Plate | Positive | Negative | |
|---|---|---|---|
| Chemical change | $Pb_2O_5$ $PbO_2$ $PbO$ | $PbO_2$ $Pb$ | The arrows show the direction of the changes in the plates during the operation |
| Charging | Oxidation<br>←<br>Anode | Reduction<br>Cathode | |
| Discharging | Reduction<br>Cathode | Oxidation<br>←<br>Anode | |

Charging makes the plates more unlike one another and discharging brings them into states more like one another.

The discharge should not be allowed to go too far. It should be stopped before either plate reaches the stage of lead monoxide, PbO, for this substance in contact with sulphuric acid forms lead sulphate. Lead sulphate has such a high resistance to the passage of electricity that the current passes it by and it takes no further part in the actions in the cell. The amount of available material on the plate is therefore reduced and the storage capacity of the cell for energy is lowered. Further, by screening the part behind it from being charged, the sulphate trouble is liable to spread and will in time put the cell out of action. Most of the precautions to be taken in the maintenance of lead accumulators are intended to prevent the occurrence of this sulphate. First and foremost the cell should not be allowed to discharge completely. Discharge should be stopped when the e.m.f. has fallen to 1·8 volts in the portable type or 1·6–1·7 in the stationary type. This does not cause any loss of capacity, for the amount of energy remaining in the cell at this stage is negligible. Second, and almost as important, the cells must not be allowed to stand discharged for any longer time than can be avoided, because in this state the acid is liable to react with the oxides on the plates and form the objectionable sulphate. Third, the acid must not have too great a strength, because this tends to favour the production of sulphate in the later stages of discharge. Fourth, the discharge should not be at too great a rate, as then it does not occur evenly, and some parts of the surface of the plates may be discharged too far and become sulphated. In addition, too rapid a charge or discharge may cause "buckling" of the plates as the different oxides occupy different volumes. This may cause the plates to touch and thus produce a short circuit, or it may loosen the oxides from the plates. Fifth, a serious overcharging should be avoided, as the evolution of gas bubbles in large quantities in the pores of the oxides is liable to loosen them from the plates. The final stage of charging should be carried out with reduced current. Sixth, owing to the gassing which occurs in the last stage of charge and to some extent during the earlier stages, there is a loss of water by electrolysis. This should be made up with pure distilled water only. This gassing is due to the fact that the oxidation and reduction at the plates does not reach completion all over at the same time, and so some parts are finished before the others.

There is no accurate method of judging the state of charge or discharge of a cell except a measurement of the ampere-hours put in or taken out. However, an estimate can be formed of the state of the cell from a measure of the density of the acid. During discharge suboxide of lead, $Pb_2O$, is formed on the negative plate. This combines with sulphuric acid from the electrolyte to form a compound called *plumbous sulphate* $Pb_2SO_4$. This is also grey, and being a conductor, does not hamper the action in any way. The electrolyte is thus weakened in proportion as the discharge goes on, and the lowering of the density of the acid, when compared with the known densities when fully charged and discharged, is a guide to the state of the cell. The acid is returned to the electrolyte during charge. The colours of the plates are not reliable as a guide to the state of the charge. The negative plate is always grey and varies little in colour. The positive plate is black when charged and light brown when discharged. But as the surface of a discharged plate turns black during the first part of the charge and the surface of a charged plate changes to brown during the first part of the discharge, the colour of the surface is no guide to the state of the bulk of the plate.

In order that the capacity of a cell should be large, a large amount of material must be available for reduction or oxidation at each plate. A metal is of a very compact nature generally, so that only the surface is available for attack. Planté found that if he used lead sheets in his cell and charged it up repeatedly, reversing the direction of the charging current each time, the amount of energy stored during a charge gradually increased. This was due to the surface layer, oxidized during one operation, not being replaced as a compact skin on the plate when it is reduced during the next operation, but remaining as a spongy layer. This allowed a further skin of lead to be oxidized, and so the amount of lead oxidized or reduced is increased step by step. This operation is called *forming* the plates. It is a costly and tedious process. It is accelerated by constructing the plates with a surface covered with narrow, thin ribs with deep, narrow grooves between to expose as large a surface as possible. The preliminary forming of these plates is carried out in a neutral solution of a sulphate of magnesium or soda, as this is found to produce a deeper

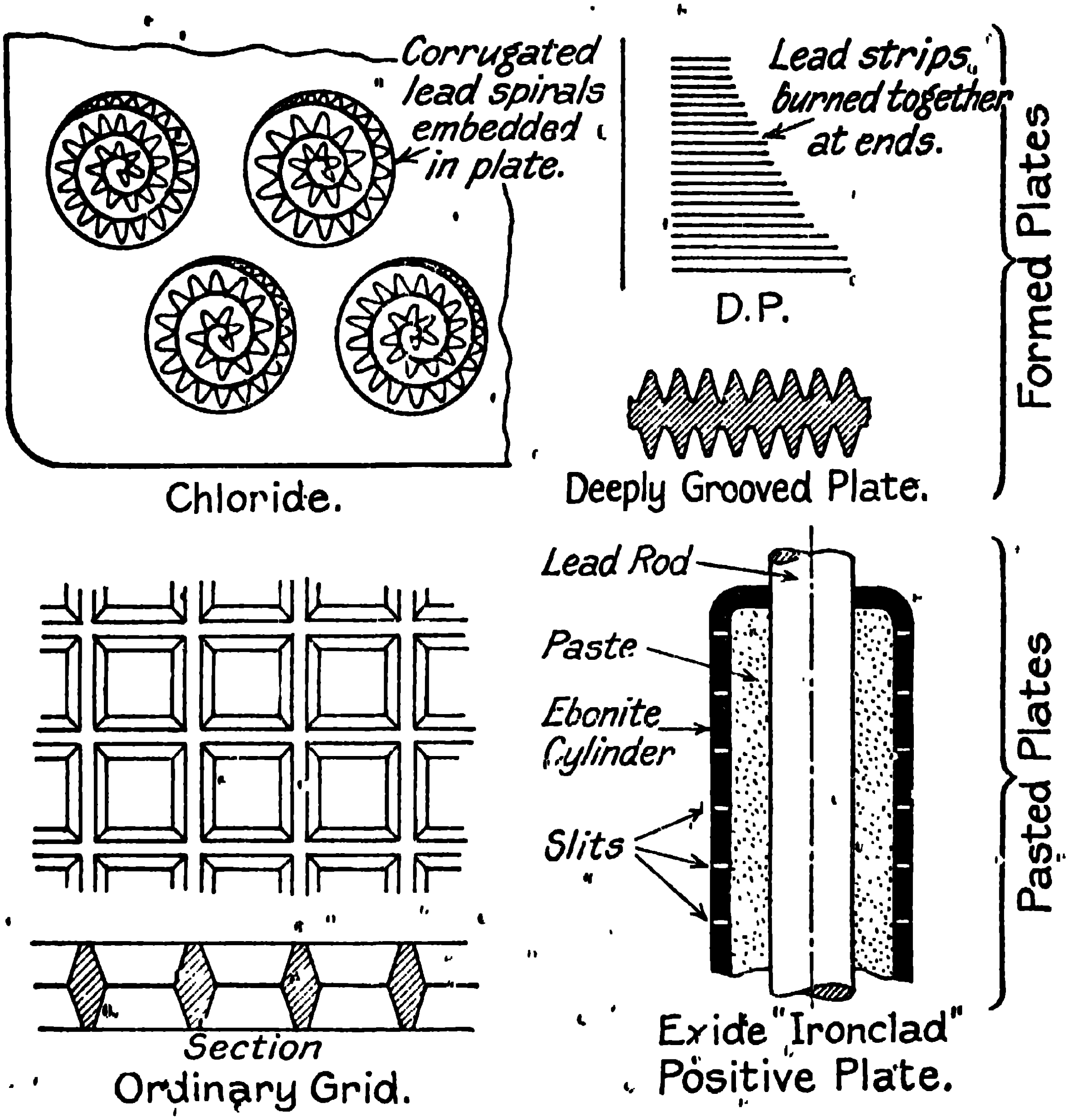

FIG. 88. PLATES IN LEAD ACCUMULATOR

porous layer than sulphuric acid. The plate is then washed free from the salt before being put into service. The other method of increasing the capacity of the plates, due to Faure, is to make them with cavities which are filled with a paste of lead oxides with some additions which have been found to improve the final state of the plate. By using the plate as an electrode with a special electrolyte, as in the case of the formed plates, these *pasted* plates have the included pellets of paste brought into a suitable spongy state and making good contact with the lead framework. Some forms of the plates used in lead accumulators are shown in Fig. 88.

The plates should be supported above the bottom of the container so as to avoid the risk of short-circuiting being caused by loose pieces of paste falling off the plates. There should be enough acid in the electrolyte in the cell to keep the strength of the solution from falling too low at the end of the discharge, and the solution should not be too strong when the cell is fully charged, as the cell will not hold its charge so well, and when discharged is more liable to sulphate. This requires a certain bulk of electrolyte. The plates are arranged alternately positive and negative, and are kept apart by insulating rods or porous separators. The capacity of the cell in ampere hours is proportional to the total active area of either set of plates. In a stationary battery, where weight is not a consideration, the containers are made ample in volume and the plates are stout to prevent buckling. On the other hand, in portable cells the ideal conditions are sacrificed to save weight and space. Only a small space is left below the plates for the accumulation of sludge, the plates are much lighter in construction, and the volume of the electrolyte is reduced, but it is more concentrated, giving a slightly higher e.m.f. The life of portable cells is thus not so long as that of the stationary pattern. However, with careful treatment, they provide much better service than primary cells at a very much lower cost. Typical charge and discharge curves are shown in Fig. 87.

(*b*) THE NICKEL-IRON ALKALINE CELL. Another type of cell was suggested by Planté, but was not satisfactorily developed into a practical form until Edison succeeded in devising his cell. He used the hydroxides of nickel in pockets in the positive plate and hydroxide of iron in pockets in the negative plate. The plates are of nickel steel pierced with small holes to allow the electrolyte to reach the pastes, and the electrolyte is caustic potash solution ($KOH$). The electromotive force when charged is 1·4–1·6 volts, falling gradually to 1·2 volts during discharge. The cells are much used in electric traction as they are not affected by mechanical shocks and vibration. The e.m.f. is not so constant during discharge as that of the lead accumulator. One of their great advantages is their ability to stand up to heavy charging rates. This makes it possible to give a useful boosting charge during a short interval of duty, e.g. a meal hour. It is not safe to do this with lead accumulators, and this property is therefore a strong point in favour of the nickel-iron cell for electric vehicles. Charge and discharge curves are shown in Fig. 89.

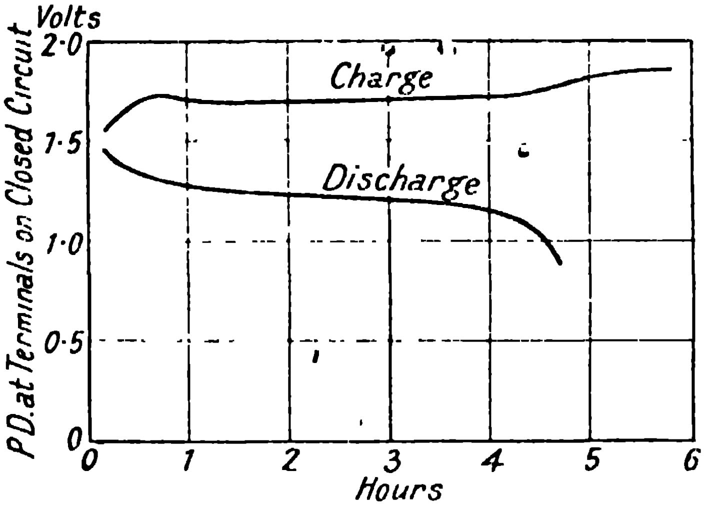

FIG. 89. CHARGE AND DISCHARGE CURVES OF NI-FE ACCUMULATOR

## EXPERIMENTS

### ON THE SUBJECT OF CHAPTER V

1. Connect up a circuit as in Fig. 84, using in the cell $C$ two copper plates, and, for the liquid, distilled water. Measure the current and p.d. Replace the distilled water by a solution of any salt or acid, and repeat the measurements. Observe the actions at the plates.

2. Connect up a circuit as in Fig. 84, so as to pass a current, measured by the ammeter $A$, through the copper sulphate solution $C$. The electrodes are clean copper plates 3 or 4 in. square, which have been weighed dry. Measure the p.d. between the plates by the voltmeter $V$ of high resistance. Pass a steady current of about 1 A. for 20 min. and measure the increase of weight of the cathode and decrease of weight of the anode. Repeat for another value of the current. Calculate the copper deposited per coulomb.

3. Continuing Expt. 2, take readings of $A$ and $V$ for as wide a range of current as possible. Plot a graph of the readings. What deductions can be made from the graph?

4. Repeat the experiments using platinum electrodes in dilute acid in the electrolytic cell along with arrangements to collect and measure the volume of the gas. Compare the graph of *A* and *V* with that obtained in Experiment 3. What deductions can be made?

5. After running the gas voltameter for some time, open the switch and observe the behaviour of the voltmeter. What is the explanation of this?

6. Construct a simple cell with plates of copper and zinc in dilute acid. Connect in a circuit as in Fig. A. Observe the readings of *A* and *V* and how they vary with time, opening the switch for a second or two every minute to read *V* with open circuit. Obtain the e.m.f. and resistance of the cell from the readings, and plot graphs of *E* and *r* on a time base. Explain the graphs.

7. Repeat, using zinc and carbon plates.

8. Repeat, using zinc and carbon plates in a solution containing chromic and sulphuric acids.

9. Test a Daniell cell in this manner.

10. Repeat for a Leclanché cell and for a dry cell, e.g. flash lamp battery.

11. Connect up a circuit as in Fig. 81, using for the electrolytic cell two lead plates in dilute sulphuric acid. Using a d.c. source of about 4 V., send a steady current of about $\frac{1}{4}$ A. through the cell. Note the readings of *V* each minute. When the cell has been running on charge for 20 min. or so, open the switch and re-connect it as in Fig. A. Read the voltmeter each minute until the cell is discharged when supplying a current about half the charging current.

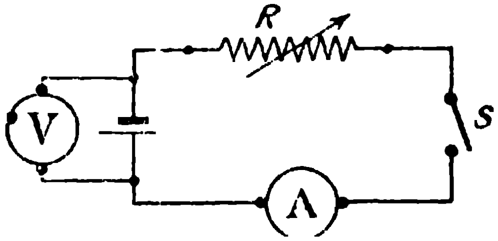

FIG. A

12. Place a solution of copper sulphate in a thermos flask. Read the temperature by a thermometer divided in $\frac{1}{10}$ deg. Drop in about 1 g. of zinc dust and measure the rise of temperature. By means of a heating coil supplied with a measured current, raise the temperature a similar amount. Calculate the joules = watt-sec. required to give the same heating as 1 g. zinc. Calculate the joules available from the electro chemical equivalent of zinc 0·000339 g. per coulomb. Compare with e.m.f. of Daniell cell.

13. Compare the e.m.f.'s of cells by means of a potentiometer.

14. Calibrate a voltmeter against a standard cell by means of a potentiometer.

15. Use a potentiometer to measure the p.d.'s at the terminals of a cell when open circuited and when supplying a measured current or

**when connected to a known resistance. Deduce the internal resistance of the cell and its e.m.f.**

**16. Calibrate an ammeter against a standard cell by means of a potentiometer.**

## EXERCISES

1. From the following data calculate the e.m.f. of a Daniell cell.

Electro-chemical equivalent of zinc, 0·000339 g. per coulomb.

Heat evolved when 1 g. zinc dust is stirred in copper sulphate solution, depositing copper = 773 cal. (1·10 V.)

2. The resistance of a fault to earth on a telegraph line is found to be greater when the leakage current through the fault is in one direction than when it is reversed. Explain this. Which direction of the current should increase the resistance of the fault? (Earth to line.)

3. Calculate the weight of lead which is used on the negative plates of a lead accumulator for a charge of 40 Ah. (155 g.)

4. Calculate the volume of mercury deposited per ampere hour in a mercury electrolytic ampere hour meter. (0·275 $\text{cm}^3$.)

5. What rules should be observed in order to maintain a battery of lead accumulators in condition? Give the reason for each precaution, and state the consequences of neglect. (See p. 170.)

## CHAPTER VI

## THE ELECTRON THEORY

1. In the first chapter it was stated that the chemical action produced by the passage of an electric current through a solution is proportional to the total quantity of electricity which has passed. This implies that each atom of a metal, copper, for example, carries the same charge of electricity. Furthermore, the charges carried by an atom of hydrogen or of silver, sodium, potassium or chlorine are all found to be identical in size; and the charges carried by atoms of copper, zinc, cadmium, mercury, and many other atoms are each double that size. Similarly, another set of atoms carry a charge which is three times the charge of a hydrogen atom. This charge seems to be a natural unit and no fractions of it appear to occur. Electricity is therefore seen to exist in grains of a definite size. The same conclusion is to be drawn from a study of the conduction of electricity in gases. It is possible to test these conclusions very thoroughly because each atom or *ion*, carrying its charge with it in its motion, carries, so to speak, a label by which it can be identified, and its point of departure and its ultimate resting place can be traced by the charges lost and gained. A comparison of these charges with the amount of material lost or collected gives a direct measure of the charge carried by each ion on its journey.

2. **Constitution of the Atom.** The conclusion to which scientists have been led by a study of all the phenomena can be stated simply: each atom is made up of a core or nucleus which has a positive charge. Round this core there circulates a system of negatively charged particles, each having the same charge; this charge is the same as that on a hydrogen ion. These particles are called *electrons*. The charge on the central core of the atom is equal and opposite to the sum of the charges of the electrons which are circulating round it, so that the atom as a whole is neutral. The atoms of different elements differ from one another in the charge on the central core, and therefore in the number of electrons which are in circulation in the outer parts of the atom. The behaviour of the atom towards

others depends on these outer electrons, as the inner + and − charges of the atoms neutralize each others' effects to a great extent. The outmost electrons may thus, in the case of some atoms, be only held very loosely. When a large number of such atoms are together, as they must be electrically neutral, the full complement of electrons must be present. The outmost electrons are then not firmly linked each to its own atom, and they may drift easily from one atom to another. A material of this constitution would be a conductor of electricity. The atoms, also, would tend to form positive ions. These two properties we have seen are general properties of metals. When in the solid state metals are good conductors; and when the atoms are separated from one another, as in solution, they form positively charged particles ions and move towards the cathode when an electric field is applied to the solution. The charge on one of these ions depends on how many electrons are easily lost from the outside of an atom. There may be a surplus of one unit charge, equal to the negative charge on an electron, but positive, or there may be two such charges, or in some cases three or more. But the charge on an ion must always be a multiple of the basic unit.. This theory of the constitution of metals agrees then perfectly with the known laws of electrolysis, as well as explaining the high conductivity of metals in bulk. Non-metallic elements consist then of atoms which do not easily lose electrons from their outer parts. They should therefore be insulators, and, in fact, they are generally very poor conductors. There are some borderline elements: carbon, silicon, boron, tellurium are examples; which conduct to a greater or less extent. They are explained naturally as having atoms which hold to their outer electrons with different degrees of obstinacy. In the case of carbon the freedom of movement depends largely on the manner in which the atoms are built together to form the solid. The diamond is quite a good insulator. Charcoal is a very poor conductor whose properties depend very much on the manner in which its preparation has been carried out. Graphite is quite a good conductor, though it is far short of possessing the conductivity of a metal. It is to be expected, if this theory of the constitution of materials is a fair picture of the real state of things, that if we apply to an insulator a gradually increasing potential difference we shall arrive at a stage when the field is strong enough to pull the outer electrons away from the atoms to which they belong, and thus to force a current through the material. The breakdown of the insulation may be sudden or gradual according to the particular circumstances. This is found to be the case in fact, and Table IV on page 46 gives the value of the dielectric strength for some typical insulators. The theory is satisfactory, therefore, in the case of insulators also as far as it goes.

The formation of compounds by the combination of atoms of different kinds is also in accordance with these ideas. To take a simple case, the metal sodium is made of atoms which form ions having a single positive charge. Chlorine is a non-metal whose atoms can take on an electron in addition to the number required to balance the positive charge on the core: it thus naturally forms negative ions possessing a single negative charge. These two ions combine together to form the compound sodium chloride, which is neutral and non-conducting in the solid state, having no free electrons. In solution however the two ions become separated from one another by the jostling of the molecules of the water, and the solution becomes a conductor of electricity, as it contains positively and negatively charged particles free to move as the electric field directs. The conductivity of the solution is proportional to the amount of salt dissolved, until the concentration is such that the ions are so numerous that they interfere with one another's freedom. The conductivity at this stage does not increase in proportion to the concentration but at a less rate.

**3. Temperature Effects.** When a substance is heated the amount of energy of the molecules is increased. This energy is mainly in the form of kinetic energy of motion—the molecules are in a state of vibration about their rest positions. The agitation is increased by a rise of temperature, and this has an effect on the electrical conductivity. In the case of a metal there is no increase in the number of conducting particles, but the increased movement of the atoms hampers the movement of the electrons as they thread their way among them. The resistance of the metal increases as the temperature rises. In the case of a poor insulator the outer electrons are only moderately firmly held, and the increased jostling loosens some of them and so improves the conductivity of the material. The increase in the number of carriers more than balances the increase in the obstruction to the traffic due to the vibration of the stationary molecules. In consequence a rise of temperature decreases in general the insulating quality of a material. As the passage of the leakage current heats the path along which it flows the breakdown of the insulation is cumulative in its effects. The temperature may rise and the conductivity increase until the insulation is destroyed. This reduction of insulating quality with rise of temperature is a general property of all substances and no material exists which will not conduct quite well at a temperature of 2 000° C.

4. **Contact Potential Difference.** A metal is a conductor because its atoms do not hold their full allowance of electrons firmly, although there is some attraction between them. This attraction varies with the metal, some metallic atoms having a greater attraction than others. If two different metals are placed in contact this difference in attraction for electrons shows itself by the setting up of a potential difference; some of the loosely held electrons transfer themselves from one metal to the other. This effect was first discovered by Volta 140 years ago. As it was very difficult to show that this potential difference really existed and that no other explanation of the effects would do, it is only recently that it has been finally settled that Volta was right and that two metals in contact take up different potentials. When different metals are joined in a complete circuit, with no e.m.f. included, all the potential steps at the junctions of the metals balance out, there is no resultant e.m.f. in the circuit, and no current flows. This is one of the difficulties in the demonstration of the effect; however, if all the circuit of metals is not at the same temperature there is an e.m.f. in the circuit. This is explained by the fact that the relation between the atom and the outer conductivity electrons is changed by temperature, or more definitely by the difference in the energy of vibration of the atoms. The potential difference between two metals at their junction will then depend on the temperature there, and if all the junctions are not at the same temperature the potential steps will not all balance out. There are in addition potential differences set up in a piece of uniform metal if it is not all at the same temperature. In a complete circuit these phenomena give rise to what is known as *thermo-electromotive forces*. These are well known and have been used for a long time for the

measurement of temperature. The contact potential difference also has to be taken into account in some electric measurements. For instance, electrostatic voltmeters are generally made with brass quadrants and aluminium "needles." The contact potential difference between these electrodes is not the same when the needle is connected to the positive side of the p.d. to be measured as when it is connected to the negative side. The deflection observed is not the same in the two cases on this account. Therefore for the measurement of steady p.d.'s care must be taken to keep that electrode positive which was positive during calibration. For the measurement of a.c. potentials this effect is eliminated because the alternations cause the reading to be the mean of the two obtained by reversing the connections of a d.c. potential difference of the same effective value. To link up the d.c. and the a.c. calibrations, therefore, readings must be taken with the d.c. potential difference connected both ways, and the mean of the two readings taken to compare with the a.c. reading. This precaution only applies to low-reading instruments as the contact p.d. is only of the order of a volt or so.

5. **Metallic Conduction.** It is worth while to consider the manner in which a metal conducts electricity. The atoms of the metal form a framework by their mutual attractive forces which hold them in fixed relative positions. The outermost electrons of an atom are not much further from the cores of the neighbouring atoms than they are from their own, and they experience some attraction from them. In consequence they are practically in neutral equilibrium and are not drawn specially in any particular direction. Any outside force can make them move as it directs. The fixed framework of the atoms has a positive charge equal and opposite to the charge of the loose electrons, and equal to the charge which would be required to deposit the atoms in electrolysis. One coulomb of electricity would deposit 0·000329 g. of copper of density 8·9 g. per cm.$^3$ This implies that the charge on the atoms forming the framework of 0·000037 cm.$^3$ of copper is 1 coulomb. The charge on the loose electrons in the same volume must be equal to this, e.g. 1 coulomb. The current is carried by the drifting of these electrons through the spaces between the atoms. If the current density is 1 ampere per mm.$^2$ then in 1 sec. the electrons drift a distance of 0·0037 cm. A movement

of 1 cm. requires nearly 5 min. This is quite a different matter to the rate at which an electric disturbance is propagated along a wire. If a battery is suddenly switched into a circuit, electrons are drained from the wire at the positive terminal and are fed into it at the negative terminal. This destroys the balance of equal and opposite charges in the wire, and the charges are equalized along it by an easing-off of the electrons from the negative end towards the positive end. Each one only moves a very short distance, but as the field produced by a very small amount of free charge is relatively enormous the operation of equalization is carried out in a very short space of time. The speed with which the disturbance is signalled along the wire approaches the velocity of light—300 km. per sec. The column of electrons in a wire behaves like an incompressible fluid in a pipe filled with sand. As fluid is pushed in at one end an equal amount leaves at the other, practically instantaneously. It may take a long time, however, for an individual particle to traverse the pipe. The law of friction is the same for the electricity as for the fluid in the pipe filled with sand: the drop of pressure in any given length is proportional to the amount passing per second. That is, Ohm's law is the law of friction of a fluid in such a pipe.

As the flow of a current along a conductor is really the flow of a stream of particles, the elementary current which should be considered is the motion of a single charged particle. Such a particle moving at right angles across a magnetic field is acted on by a force at right angles to the direction of the flux in the field and to the direction of motion of the particle. The strength of the force is equal to $HeV/10^8$ dynes.* A conductor which is moving across a magnetic field, the flux in which is at right angles to its length, is carrying electric charges with it in its motion. If the direction of motion of the conductor is at right angles to the flux there is a force on each unit of charge in the wire equal to

$$HV/10^8 \text{ dynes},$$

and the work done by this force in moving 1 coulomb along a length $l$ of the conductor is

$$HVl \text{ joules}.$$

Now joules per coulomb gives the measure of the e.m.f. in volts. This is the law of the electromotive force induced in a conductor moving across a magnetic field, deduced from the law of force on a charge moving in a magnetic field.

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*e = charge of electron = $1{\cdot}591 \times 10^{-19}$ coulomb.

6. **Magnetism.** The electron theory of the constitution of matter gives a direct explanation of the existence of magnets. Over a century ago Ampère suggested that the magnetism of iron was due to little currents circulating in the atoms. According to the electron theory the electrons are circulating around the central positively charged nucleus; thus they provide the little atomic currents of Ampère. An electron travelling round an orbit is easily seen to be equivalent magnetically to a current flowing round the line of the orbit. It would thus be the source of a magnetic flux permanently linked with the atom. This would have, in consequence, a magnetic moment. The problem is: why are not all atoms magnetic? The answer is that as in general the electrons are not all circulating in the same direction in the atom, the resultant magnetic moment is generally negligible as the orbits tend to pair off and thus cancel each other's effects. Only in a few exceptional cases is this balance incomplete. If an atom were placed in a field of magnetic flux the motions of the electrons in their orbits would be affected by the flux and the orbit would tend to turn round and change in size so as to keep the flux linked with the orbit constant. The effect of a large number of such atoms forming a piece of material would be to make the flux density in the space occupied by the matter the same as it would have been if the matter were not there. In the exceptional cases of the magnetic elements there is some hindrance to this automatic adjustment of the orbits and this produces their magnetic qualities. In the other cases there is merely a trace of the property which causes sometimes a very slight increase of the resultant flux, and in others a very slight decrease. These elements are termed *paramagnetic* and *diamagnetic* respectively. Some metals which are nearly related to the magnetic metals, iron, nickel and cobalt, show similar magnetic behaviour to them in some of their alloys. Thus the metal manganese when alloyed with silver or one or two other metals is magnetic like iron. This property was first discovered by Heussler in some alloys of manganese with copper and aluminium.

7. **Emission of Electrons.** The electrons which can move

freely in the interior of a metal do not escape from the surface unless there is some agent to assist their passage through it. They can move about inside because they are pulled equally in all directions by the positively charged nuclei all around them, but to get out requires work to be done against this attraction. This work can be supplied in different ways, and three of these are of practical importance technically. They are *thermionic emission*, *photo-electric emission*, and emission due to the arrival of a high velocity electron or positive ion (*secondary emission*). These phenomena are best studied in a vacuum as the air or other gas present would hinder the motion of the electrons, as well as adding another complication due to the fact that when an electron is moving sufficiently fast through a gas and impinges on an atom it may ionize it, that is, it may break up the atom into a positive ion and an electron. This ionization of the gas masks the passage of the electrons from the surface unless the pressure of the gas is very low, so that the ionization can be ignored.

(i) PHOTO-ELECTRIC EMISSION. This will be considered first as it is the simplest to describe. If a glass bulb is pumped to as high a vacuum as possible after there have been sealed into it two electrodes of suitable metals, and one of the electrodes is illuminated by a beam of light, it will be found that a discharge of negative electricity can be obtained from the illuminated electrode. If the p.d. between the electrodes is sufficient to cause all the electrons emitted to be carried across to the other electrode, the current will be found to be proportional to the intensity of the light beam. The size of the current will depend mainly on two things: the metal of which the illuminated electrode is composed and the colour or *wavelength* of the light. The most sensitive metals are those which are most electropositive. Photo-electric cells have been constructed whose sensitiveness to colour is not very different from that of the normal eye; these are used in lamp factories in the inspection of the completed lamps. Another type is used in sound film work. Others which are sensitive to rays which are invisible to the eye can be employed to detect the passage of unauthorized persons. These cells have also been used to operate relays to switch on and off a lighting system according to the brightness of the light from the sky. If this is below a certain standard, the lights are switched on and when the lighting is no longer needed they are switched off. A considerable economy is obtained as the lights are never on too long, and they come on automatically in an emergency such as a fog, without the employment of a special staff. Photo-electric cells are of two types, *vacuum* and *gas-filled*. Both types have the same electrodes but the vacuum cells are pumped out to as high a vacuum as possible. The passage of electricity between the electrodes consists solely of the electrons liberated from the anode. In the gas-filled cells some gas is allowed to remain in the bulb. When electrons are set free from the anode they are accelerated by the electric field between the electrodes and ionize by collision the gas through which they pass. The number of carriers is thus increased and a larger current passes through the cell than that due merely to the liberated electrons. The gas-filled type, while more sensitive than the vacuum type, cannot stand such a high p.d. without flashing over.

(ii) SECONDARY EMISSION. Just as an atom of a gas is ionized by the impact of an electron or ion if it arrives with sufficient energy, so the surface of a metal emits electrons under the bombardment of electrons. The incident electrons provide the energy needed to get the contained electrons through the surface. If an electric field is maintained between two electrodes in a space filled with a gas at low pressure, a steady discharge may be maintained in the following manner. An electron moving towards the anode, driven by the field, will ionize an atom with which it collides if the field is sufficiently intense. This provides two electrons, instead of one and a positive ion. These moving in obedience to the field produce further ions and electrons by collision, and the conductivity of the space is high. To maintain this state of affairs a supply of electrons entering the space is required. This is provided by the impact of positive ions on the cathode producing a secondary emission of electrons. The discharge is thus able to provide the condition for its own maintenance if the potential difference is great enough. There is a difficulty about starting the discharge, but once started the discharge is kept going. The striking potential difference is higher than the running potential difference. Gas discharge tubes are widely used for illuminated signs and for street lighting. To avoid the necessity for a high p.d. to start the discharge; a heated filament is often used to provide the initial supply of electrons.

THERMIONIC EMISSION. The energy needed to get the electrons through the surface of a metal into the space outside can be drawn from the heat energy of the metal itself if it is at a sufficiently high temperature. This emission was in fact the first case to be observed and studied. The emission may reach a very high value amounting to some amperes per square centimetre. It increases very rapidly with rise of temperature and, conversely, decreases very rapidly with fall of temperature, becoming quite negligible at ordinary temperatures. The emission varies greatly from different materials, and also with the state of contamination of the surface. The oxides of the alkaline earth metals are found to give a very high emission if they are placed on a heated surface, thus forming what are called *Wehnelt cathodes*, after their discoverer. One of these metals is thorium, and it was found that if a tungsten wire was alloyed with thorium sufficient electrons were obtained from it at a much lower temperature than was possible from a pure tungsten wire. The pure tungsten needs to be heated so as to glow brightly before the emission is great enough to be of use. The filaments used are therefore classed as *bright emitters* and *dull emitters*, which may be coated or thoriated filaments. Lately for small discharge tubes an electrode of the oxides of earth metals has been used which is heated indirectly by a wire through which an electric current is passed. This construction avoids some difficulties in use

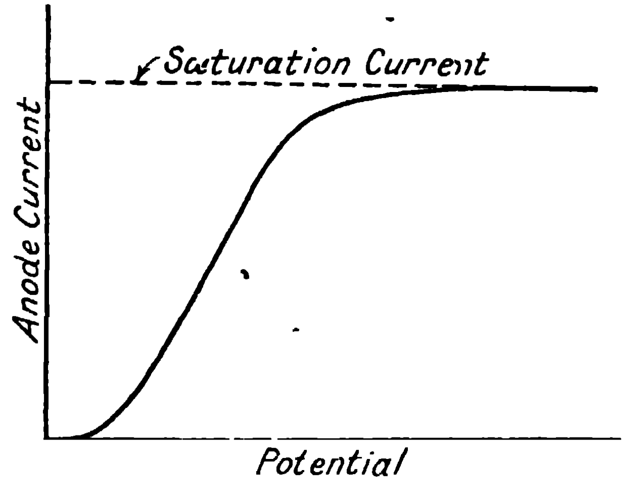

FIG. 90. PHOTO-ELECTRIC CELL AND DIODE CHARACTERISTIC

which arose from the potential drop along the filament when heated directly by a current.

When one of these electrodes capable of emitting electrons is enclosed in an evacuated bulb with another electrode we have a *diode*. If the second electrode is at a negative potential with respect to the emitter, no current can pass, for there are no carriers available to convey it across the space. The electrons are driven back on the electrode from which they came.

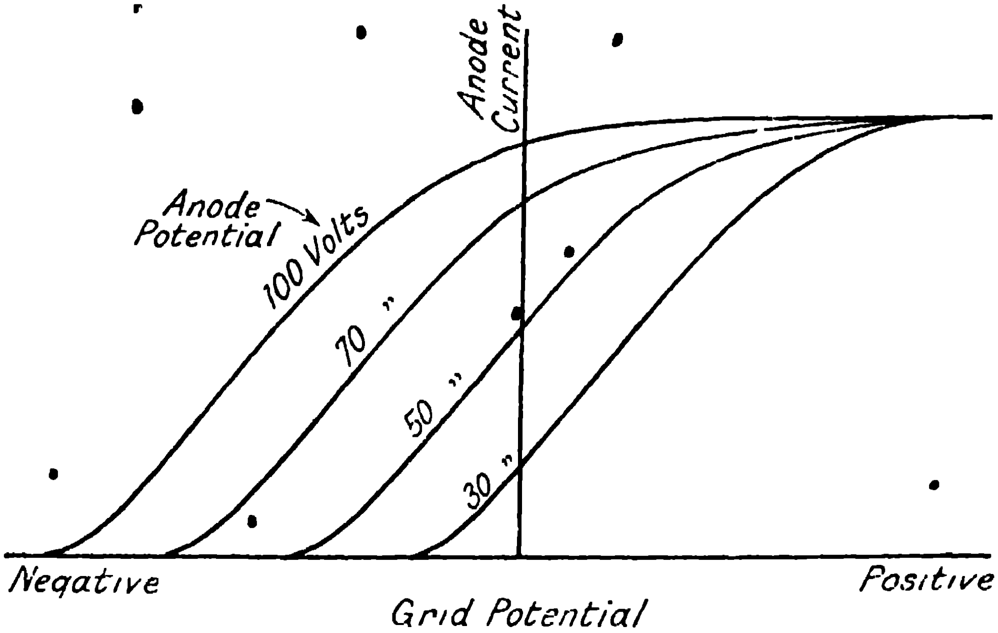

FIG. 91. TRIODE CHARACTERISTICS

If the second electrode is positive, however, a current can and does pass across. The electrons are drawn away from their parent electrode and carry the current to the anode. This arrangement acts as a rectifier if an alternating p.d. is applied between the two electrodes. The current depends on the p.d. between the electrodes. All the electrons emitted do not travel across to the anode because they are hampered by the other electrons which are between them and that electrode. These being negatively charged repel their followers back towards the cathode and some return to it. Only if the p.d. is sufficiently high do all the emitted electrons travel across. The current then has its largest possible value, known as the *saturation current*. Now if another electrode is placed in the space between the other two its potential may be adjusted to help or to hinder the passage of the electrons across. This forms the *three electrode valve* or *triode*. The name "valve" was given to the arrangement of electrodes in a vacuum because the current could only pass one way. It is found that, so far as the current to the anode is concerned, a given change of potential of this control electrode or *grid* has the same effect as a much larger change in the potential of the anode. The ratio of the two changes of potential which give the same change of current is called the *amplification factor* of the valve. This quantity is a measure of the effectiveness of the valve for amplification of small potential differences. The variation of the current to the anode or *plate* can be best shown on a diagram of the *characteristics* of a valve, showing the plate current as dependent on the potential of the grid for various fixed values of the potential of the plate. An increase of the plate potential moves the characteristic curve as a whole so that the same plate current now occurs for a value of grid potential which is less than the previous value by a fixed amount. The ratio of the change in plate potential to the corresponding change in grid potential is the amplification factor. The slope of the curves in the straight part gives the *mutual conductance* of the valve. It is the change of the plate current for one volt change in potential of the grid. It is generally stated in milliamperes per volt. Modern valves are pumped out to the greatest vacuum obtainable, as it is found that traces of gas render the characteristics of a valve unstable and variable. Gas-filled rectifiers are however used as they are capable of dealing with heavier currents than the vacuum type with smaller energy losses owing to the smaller potential drop across the electrodes. Of this type are the *mercury arc rectifiers* which have been made up to sizes which can deal with large numbers of kilowatts. These are diodes. The three-electrode type, in which the passage of the discharge can be forbidden or allowed by the third electrode has also been much developed for a variety of purposes under the name of *thyratron* or *gas relay*.

Valves with many electrodes have been designed and used, but the principles of their operation are fundamentally the same as those of the triode.

# APPENDIX ON UNITS

*(Mainly for Teachers)*

THE intelligent student is sure to ask about the units used for current and potential difference, and it is as well to summarize the situation and show how the way in which the subject has been developed in this book may be linked up with the historic or classical method.

The definitions of the units of current and p.d. given in the first chapter are the International definitions and appear to be quite arbitrary. Yet it appears in Chapter III that the force on a conductor in a magnetic field is $HIl/10$ dynes, a nice even number. Why is this? While on the other hand, the capacitance of a parallel plate condenser is

$$A/(4\pi d \times 9 \times 10^{11}) \text{ farads}$$

The fact is, of course, that nothing really is known of electricity except its effects, and the only fundamental units are those of length, mass and time.

In the book the fundamental relationships of electromagnetics are that field strength is proportional to $NI/l$, and that the force on a current is proportional to $HIl$. These are regarded as being established experimentally (Chap. III). There is also the purely electrical relationship that the work done in moving electricity is proportional to $VQ$ (Chap. I). Let the constants of proportionality in these fundamental relationships be $k_1$, $k_2$ and $k_3$ as follows—

$$H = k_1 NI/l$$

$$F = k_2 HIl$$

$$W = k_3 VQ$$

From these the induced e.m.f. equation follows (Chap. IV). If a conductor is moved a distance $dx$ in a magnetic field and in consequence a current $I$ flows under an induced e.m.f. $E$,

$$Fdx = k_2 HIldx = k_3 EIdt$$

is the work equation, $k_3$ appearing because $Idt \doteq Q$ and $E$ is measured in the same units as $V$. From this

$$E = \frac{k_2}{k_3}\frac{dHA}{dt} = \frac{k_2}{k_3}\frac{d\Phi}{dt}$$

where $A = lx =$ area swept through.

$k_1$, $k_2$ and $k_3$ can be given any values in building up a system of units. Some limitation will however be imposed if force and work are to be measured in well recognized mechanical units. A starting point is defined by deciding that force is to be measured in dynes and work in ergs.

The total flux in a long, thin solenoid of $N_1$ turns, length $l_1$ cm., area $a$ cm.$^2$ and carrying current $I_1$ is

$$\Phi = k_1 \frac{N_1 I_1}{l_1} a \text{ lines}$$

and the field strength at a point $P$ distance $r$ cm. from the end is

$$H_p = \frac{k_1}{4\pi r^2} \cdot \frac{NI_1}{l_1} a \text{ lines per cm.}^2$$

$r$ being large compared with the thickness of the solenoid but small compared with its length. The force on a conductor of length $l_2$ cm. carrying a current $I_2$ at right-angles to the field at $P$ is

$$\begin{aligned} F_p &= k_2 H_p I_2 l_2 \\ &= \frac{k_1 k_2}{4\pi} \frac{Na}{r^2} \frac{l_2}{l_1} I_1 I_2 \\ &= \frac{k_1 k_2}{4\pi} \frac{Na}{l_1} \frac{l_2}{r^2} I_1 I_2 \text{ dynes.} \end{aligned}$$

Now make $Na/l_1$, $l_2$ and $r$ each unity, and $I_1 = I_2 = I$, then

$$F_p = \frac{k_1 k_2}{4\pi} I^2 \text{ dynes.}$$

From this expression $I$ can be defined electromagnetically, in terms of a force produced. The product of the constants $k_1$ and $k_2$ is fixed.

In the system of units used in the book, which is the International or Practical system, $k_1 k_2/4\pi = 1/100$, so that $F_p = I^2/100$, or unit current is that current which, flowing in a long "unit" solenoid (i.e. one having the product turns per centimetre times cross-sectional area equal to unity) and in a conductor of one centimetre length situated one centimetre distant from one end normal to the field, experiences a force of one hundredth of a dyne. This current is called an *ampere*; the amount of silver deposited in unit time by this current, used at the outset to define it, can be determined experimentally.

In the practical system also, $k_2$ is made 1/10, so that $F = HIl/10$ dynes. $k_1k_2$ and $k_2$ are fixed therefore quite arbitrarily, but so as to make the forces arising "nice round numbers." $k_1$ follows

$$k_1k_2/4\pi = 1/100; \; k_2 = 1/10$$

$$\therefore \qquad k_1 = 4\pi/10$$

Current and field strength are now defined. Potential difference is defined as the work done when unit quantity flows, and the unit of work is taken as the joule in the practical system. Thus, since a joule is $10^7$ ergs,

$$k_3 = 10^7$$

and

$$E = \frac{k_2\,d\Phi}{k_3\,dt} \quad 10^{-8}\cdot\frac{d\Phi}{dt}$$

This unit is called the *volt*, and it is in terms of this definition that the e.m.f. of the standard cell used for the initial definition is found.

There is another system of electromagnetic units known as the *absolute* or *c.g.s.* (centimetre-gramme-second) system. In this $k_1 = 4\pi$, $k_2 = 1$ and $k_3 = 1$. Unit current is 10 amperes, and unit p.d. or e.m.f. is $10^{-8}$ volt; it follows that the unit of resistance is $10^{-9}$ ohms. This absolute system is used in the classical theory.

In the classical theory of electromagnetism, the starting-off point is the force between two magnetic "charges" called *poles*. The inverse square law relating to the force between two poles is stated and proved experimentally, and unit pole strength is defined as that pole which at a distance of one centimetre from an equal pole experiences a force of one dyne. Then unit field strength is defined as that field in which a unit pole will experience a force of one dyne. Finally, unit current is defined as that current which, flowing in one centimetre length of the arc of a circle of one centimetre radius, produces unit field strength at the centre of the circle. Using these definitions and using also Ampere's law and Biot and Savart's law, the whole of the results arrived at in Chapters III and IV are obtained. Ampere's law (sometimes attributed to Laplace) states that the contribution of the current $dI$ in a filament $ACB$ of length $dl$ to the magnetic field at $P$, distant $r$ from the centre $C$ of the filament, the angle $PCA$ being $\theta$, is

$$dH = dI.dl.\sin\theta/r^2$$

the direction of the field being at right-angles to the plane containing $P$ and $ACB$.

Biot and Savart's law states that the filament $ACB$ if situated in a field $H$ experiences a mechanical force

$$dF = HdI.dl.\sin\theta$$

at right-angles to the plane containing the filament.

The argument of the book will be completely linked up with that of the classical theory if the field at the centre of a circular current carrying conductor can be found. To do this it is first necessary to show that the force on the end of a long solenoid situated in a field of strength $H$ and direction along the axis of the solenoid is given by $naIH$, where $n$ is the number of turns per centimetre length of the solenoid and $I$ is the current in absolute c.g.s. units.

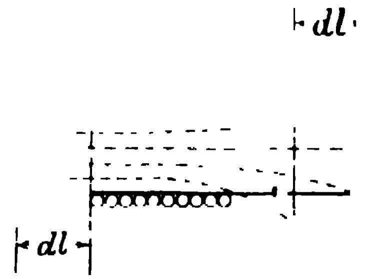

FIG. 92. FORCE ON SOLENOID IN UNIFORM FIELD

Let the solenoid be moved a length $dl$. Then the work done is $Fdl$ and this must be equal to the work estimated from the flux cutting the turns of the solenoid (Fig. 92). Over a length $dl$ the turns cut are $ndl$ and the change of linkages is $n.dl.d\Phi$ where $d\Phi$ is that part of the total flux $\Phi$ which leaves the solenoid in the length $dl$. Since the current is $I$, the work done in the length $dl$ is $nIdl.d\Phi$ and the work done over the whole length of the solenoid, i.e. for a change of flux from $\Phi$ to zero is $nIdl.\Phi$. Thus

$$Fdl = nI\Phi dl$$

and

$$F = nI\Phi = naIH$$

which was to be proved.

Now let the solenoid current be $I_1$ and the current in a circular conductor of radius $r$ situated at the end of the solenoid as in Fig. 93 be $I_2$. The field at any point $P$ on the circle due to the current in the solenoid is $\Phi/4\pi r^2$, i.e.

$$H_p = nI_1a/r^2$$

and the force $F$ on the circle is in consequence

$$\begin{aligned} F &= 2\pi rI_2nI_1a/r^2 \\ &= (2\pi/r)I_1I_2na \end{aligned}$$

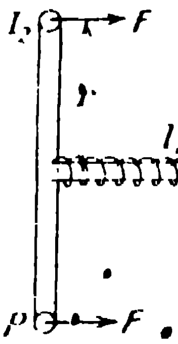

FIG. 93. FORCE BETWEEN COIL AND END OF SOLENOID AT CENTRE

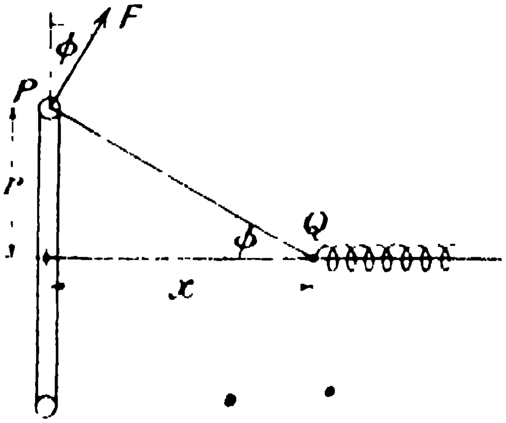

FIG. 94. FORCE BETWEEN COIL AND END OF SOLENOID ON AXIS

and since action and reaction are equal and opposite, this must be equal to the force on the end of the solenoid in the field $X$ due to the current $I_2$ in the circle. This force is $naI_1X$ and so

$$naI_1X = (2\pi/r)I_1I_2na$$

or

$$X = (2\pi/r)I_2$$

Thus the field strength at the centre of a circular current of unit radius is $2\pi I$, and if the current is unity and the current flows only in an arc of unit length instead of through the whole circumference, the force is also unity. The classical definition of unit current has been reached.

Another expression which is arrived at very simply from Ampere's law is that for the field at any point on the line drawn through the centre of the circle perpendicular to its plane. Let (Fig. 94) $x$ be the distance from the centre to the point $Q$, at which the end of a solenoid is placed. Then the force on the circle will be normal to the line joining $P$ and $Q$ and making an angle $\phi$ with the axis, and the component in the direction of the axis will be $F \sin \phi$. The component tending to move the circle is, since the field at $P$ is $4\pi n I_1 a/4\pi(x^2+r^2)$,

$$F \sin \phi = \left[\frac{n I_1 a I_2 . 2\pi r}{(x^2+r^2)}\right] \sin \phi$$

$$= \frac{2\pi n a r^2 I_1 I_2}{(x^2+r^2)^{\frac{3}{2}}}$$

and if $X$ is the field at $Q$ due to the current $I_2$ in the circle, $F \sin \phi$ is also $n a I_1 X$.

Thus

$$X = 2\pi \frac{r^2}{(x^2+r^2)^{\frac{3}{2}}} . I_2$$

In the classical theory there is another system of units altogether, the electrostatic c.g.s. system. In this, charge is defined from the inverse square law of electric quantity as that quantity which at a distance of one cm. from an equal quantity experiences a force of one dyne. This definition also determines the definition of unit current (unit quantity per second) and p.d. follows by making $k_3 = 1$, i.e. when unit quantity is moved between two points where unit p.d. exists, the work done is one erg.

It is found experimentally (or theoretically in more advanced theory) that one coulomb = $3 \times 10^9$ e.s. units of quantity, or one c.g.s. unit = $3 \times 10^{10}$ e.s. c.g.s. units; and it is of very great interest that $3 \times 10^{10}$ cm. per sec. is the velocity of light. It follows, in order that $k_3$ shall be $10^7$ in the practical e.m. system, that 300 volts = one e.s. unit of p.d.

Unit electric field is one in which unit charge experiences a force of one dyne.

Electric flux density is not, in the e.s. system, equal to the force on a unit charge, i.e. it is not identical with field strength. Instead, one line is taken to arise from every unit charge.

Thus a charge $Q$ units on a small sphere has $Q$ lines spreading out from it, and at a radius $r$ cm. the flux density $D$ is $Q/4\pi r^2$. The force $P$ on a unit charge is, however, from the definition of charge and from the inverse square law, $Q/r^2$. Thus $P = 4\pi D$; the electric field strength is $4\pi$ times the flux density. Electric field strength is also measured by the slope of the p.d.-space curve. Its unit in the practical system is one volt per cm.

Using e.s. c.g.s. units, if the field strength between the plates of a parallel plate condenser charged with $Q$ units is $P$, the flux density being $D = Q/A$ and the p.d. between the plates being $V = Pd$,

$$V = Pd = 4\pi Dd = 4\pi Q(d/A)$$

and the capacitance in e.s. units is

$$C = Q/V = A/4\pi d$$

If $Q'$ is the charge in coulombs and $V'$ the p.d. in volts, then $Q = 3 \times 10^9 Q'$ and $V = V'/300$, and the capacitance in farads is

$$C' = \frac{Q'}{V'} = \frac{Q}{3 \times 10^9} \times \frac{1}{300V} = \frac{1}{9 \times 10^{11}} \times \frac{Q}{V}$$

$$= \frac{A}{4\pi d} \times \frac{1}{9 \times 10^{11}}$$

This result was arrived at in Chapter II, from the statement that the force in dynes between two charges $Q$ and $q$ coulombs is $(Qq/r^2) \times 9 \times 10^{18}$. The flux unit adopted was such that from each coulomb $4\pi \times 9 \times 10^{18}$ lines proceeded, so that field strength (force in dynes on one coulomb) was measured by flux density. If field strength had been measured by the force in dynes on one e.s. unit, each coulomb would have given rise to $4\pi \times 3 \times 10^9$ lines in order that field strength should be measured by flux density; and each e.s. unit would give rise to $4\pi$ lines. This was done to make the treatment of the electric and magnetic fields the same. This difference of the flux unit adopted in the book from the usual flux unit of the classical theory (one line to each unit charge) should be noted.

Using the e.s. unit of current and potential, but retaining the e.m. unit of field strength, $k_1$ and $k_2$ will each be reduced by $3 \times 10^{10}$ compared with the e.m. c.g.s. system, while $k_3$ is still unity by definition. The various values of $k_1$, $k_2$ and $k_3$ may be collected together as follows—

| Constant of Proportionality | E.M. Units | | E.S. Units* |
|---|---|---|---|
| | Practical | C.G.S. | C.G.S. |
| $k_1$ . . | $\frac{4\pi}{10}$ | $4\pi$ | $\frac{4\pi}{3} \times 10^{-10}$ |
| $k_2$ . . . | $\frac{1}{10}$ | 1 | $\frac{1}{3} \times 10^{-10}$ |
| $k_3$ . . . | $10^7$ | 1 | 1 |

* But field strength measured in e.m. units.

# INDEX

ACCUMULATOR, 4
——, lead, 168
——, ——, maintenance of, 170
——, ——, plates in, 172
Air-core, 83
Alternating current, 4, 12
—— —— generator, 125
—— ——, inductance in, 137
—— ——, p.d. across condenser, 52
—— e.m.f., 122
—— p.d., 52
—— voltage, 52
Alternator, 4, 125
Ammeter, 20, 104
——, calibration of, 12, 35
——, hot-wire, 11
——, moving iron, 107
—— shunt, 28
Ampere, 5
—— balance, 92
Ampere-hour, 5
Ampere, legal, 5
Ampere's Law, 88
Ampere-turn, 84
Amplification factor, 188
Anode, 3, 152
Appendix on units, 189
Atom, constitution of, 177
Attracted disc electrometer, 72

BACK e.m.f., 156
Ballistic galvanometer, 14
Barretter, 19
Bichromate cell, 160
Bifilar winding, 130

CADMIUM cell, 164
Calibration of ammeter, 12, 35
—— of voltmeter, 35
Caloric, 9
Capacitance, 45, 49, 76
—— in parallel with resistance and inductance, 143
—— in series with resistance and inductance, 142
Capacitance, measurement of, 49
—— and resistance in parallel, 60
Cathode, 3, 152
Cell, 4
——, bichromate, 160
——, cadmium, 164
——, closed circuit, 163
——, Daniell, 162
——, dry, 162
——, Latimer Clark, 163
——, lead-acid, 167
——, Leclanché, 161
——, nickel-iron alkaline, 173
——, open circuit, 163
——, primary, 152
——, secondary, 165
——, Weston, 164
Charge, 43
—— curve, 54
—— and discharge curves of lead accumulator, 169
Circuit, earth, 32
——, magnetic, 99
Class experiments, 35, 76, 113, 146, 163, 174
Coefficient of mutual induction, 127
Coercive force, 94
Coil, induction in, 116
——, primary, 116, 125
——, secondary, 116, 126
Coils with current in opposite directions, 81
—— —— in same direction, 81
Commutator, 125
Complete circuit, 2
Condenser, 42
—— in A.C. circuit, 50
——, energy stored in, 48
——, insulation, resistance of, 75
—— lead, current in, 51
Condensers in parallel, 47
—— in series, 47
—— in a series with a resistance, 58

Condensers, water analogy of, 42
Conduction, metallic, 181
Conductor, 2, 178
Conductivity, 3, 15
Connection, Parallel, 13
——, series, 13
Contact p.d., 180
Coulomb, 5
Coulomb's Law, 69
Current, 35
—— in condenser load, 51
—— curve, 54
—— density, 30
—— in galvanometer, 51
—— p.d. and power curves, 139
—— resonance, 144
Curve, charge, 54
——, current, 54
——, p.d., 54
Curves, charge and discharge of lead accumulator, 169
—— of nickel iron alkali cell, 4, 175
——, current p.d. and power, 139
——, susceptibility, 99

DANIELL cell, 162
Declination, 103
Depolarization, 161
Diamagnetic, 183
Dielectric constant, 45, 70
—— constants, table of, 46
—— strength, 46
Dielectrics, 15
Diode, 187
Dip, magnetic, 103
Direct current, 4
—— —— circuit, inductance in, 171
Dry cell, 162
Dynamo, 4

EARTH magnetic field, 111
——, resistance, 29
—— return, 29
—— —— circuit, 29
Electric charge, 63
—— current, 1
—— field, 66
—— flux, 45, 66
Electrical pressure, 7
Electrochemical equivalent, 5
Electrode, 30, 152
Electrolysis, 2, 3, 152
—— of water, 6
Electrolyte, 3
Electrolytic meter, 154
Electrometer, attracted disc, 72
——, multicellular, 73
——, quadrant, 73
Electromotive force, 66, 115, 158
Electron, 177
Electrons, emission of, 183
Electroscope, 64
Electrostatic voltmeter, 71, 74
E.M.F., alternating, 122
——, back, 156
——, induced, 127
—— in rotating coil, 122
Emission of electrons, 183
Emitters, bright, dull, 186
Energy, 10
—— in magnetic field, 133
—— stored in condenser, 48
—— wasted in hysteresis, 136
Exercises, 39, 77, 113, 150, 176
Experiments, class, 35, 76, 113, 146, 174

FARAD, 45
Faraday's laws of induction of currents, 116
Faure, 172
Fields, electric, 66
——, magnetic, 79
Flux density, 116
——, electric, 45, 66
——, leakage, 101
—— linkage, 118
——, magnetic, 103, 116
Forces, coercive, 94
—— on straight conductor, 86
Forming, 171
Friction, 9

GALVANOMETER, 22, 106
——, ballistic, 44
——, current in, 51
Gassing, 168
Gauss, 84
Generator, alternating current, 125
Gold leaf electroscope, 64
Grid, 188

Henry, 127, 129
Heussler alloys, 98, 183
Horse-power, 9
Hot-wire ammeter, 11
Hysteresis, energy wasted in, 136
— loop, 94, 111
—, magnetic, 95

Impedance, 59, 77, 138
— and phase angle by 3 voltmeter method, 148
Induced e.m.f., 118, 127
Inductance, 102, 125
—, capacitance and resistance in series, 142
— in a.c. circuit, 137
— in d.c. circuit, 131
—, energy stored in, 134
— in parallel, 141
—, self, 128
— in series, 140, 142
— with mutual inductance, 145
— and resistance in parallel, 141
— — in parallel with capacitance, 143
— — in series, 140
Induction in coils, 116
— in a straight conductor, 118
Inductive circuit, growth of current in, 131
Instrument, moving coil, 104
— moving iron, 106
Insulation, 2
— resistance of condenser, 75
Insulator, 2, 179
Intensity of magnetization, 97
Internal resistance, 158
Ion, 3, 153, 177, 178

Joule, 9

Latimer Clark cell, 163
Lead accumulator, 168
Lead-acid cell, 167
Leakage flux, 101
Leclanché cell, 161
Left-hand rule, 88
Legal definition of the standard volt, 164
— standard of resistance, 21
Lenz's law, 118
Line faults, 32
Lines per cm.$^2$, 84
Local action, 163

Magnet, permanent, 102
—, polarizing, 110
Magnetic circuit, 83, 99
— dip, 103
— field of circular coil, 81
— field, 79
— —, energy in, 133
Magnetic field of solenoid, 82
— — of straight conductor, 80
— — of toroid, 82
— flux, 103, 116
— hysteresis, 95
— pole, 110
— properties, table of, 98
— pull, 108
— saturation, 95
— shunt, 105
Magnetism, 183
Magnetization, 93
—, intensity of, 97
—, residual, 94
Magnetizing field, 97
Magnetometer, 111
Magneto-motive-force, 85
Maintenance of lead accumulator, 170
Measurement of capacitance, 49
Meters, electrolytic, 154
Microfarad, 50
Moving coil instrument, 104
— iron instrument, 106
Multicellular electrometer, 73
Mutual conductance, 188
— inductance, 125, 150
— induction, 126
— —, coefficient of, 127

Negative, 4
Nickel-iron alkaline cell, 173
— — —, charge and discharge curves, 174
Non-conductor, 2

Ohm, 11
— legal, 21
Ohm's Law, 9, 11, 182

Open circuit cell, 163
Oxidation, 153

PARALLEL connection, 13
—— plate condenser, 70
—— resonance, 144
Paramagnetic, 183
Pasted plates, 173
Permanent magnet, 102
Permeability, 99
Permittivity, 45
Phase angle, 59, 141
Photo-electric cells, 184
—— —— emission, 184
Plante, 166
Plates in lead accumulator, 172
——, pasted, 173
Polarization, 160
Polarizing magnet, 110
Pole, magnetic, 110
Poles, 104
Positive, 4
Post Office box, 23, 24
Potential, 31
—— difference, 7, 12, 20, 26, 38, 115
P.D. across condenser, 51
—— —— curve, 54
Potential divider, 26
—— —— load, 28
Potentiometer, 26 28
Power, 9
—— factor, 62
—— —— correction, 144
Primary cell, 152
—— coil, 116, 125
Pull, magnetic, 108

QUADRANT electrometer, 73
Quantity of electricity, 4

REACTANCE of condenser, 57
—— of coil, 138
Receiver telephone, 109
Rectifiers, 18[illegible]
Reduction, 153
Relation of e.m.f. and flux, 124
Relay, 103
Reluctance, 85, 100
Residual magnetization, 94
Resistance, 2, 9, 11, 35
—— box, 23
Resistance of contact, 28
——, earth, 29
——, increase of, 17
——, internal, 158
——, legal standard of, 21
—— in parallel, 13, 141
—— in series, 13, 140, 142
Resistivity, 3, 15
Resistor, 13
Resonance, 142
Retentivity, 94
Rheostat, 13, 25
——, sliding contact, 25
Right-hand rule, 121
Root mean square value (r.m.s.), 12

SATURATION, magnetic, 95
Secondary cell, 165
—— coil, 116, 126
—— emission, 185
Self inductance, 128
Series connection, 13
Short-circuited, 32
Shunt, magnetic, 105
Sliding contact rheostat, 25
Solenoid, 82
Standard volt, legal definition of, 164
Straight conductor, force on, 86
—— conductor, induction in, 118
Susceptibility, 98
—— curves, 99

TELEPHONE receiver, 109
—— relay, 108
Temperature coefficient, 18
—— —— of resistance, 16
—— effects, 179
Thermionic emission, 186
Thermo e.m.f., 180
Three voltmeter method, 148
Thyratron, 188
Time constant, 132
Toroid, 82
Torque on coils, 88
Triode, 187

UNITS, appendix on, 189
Unit tube, 50

VARLEY loop test, 34
Vector, 60
—— diagram, 56, 58, 129
Volt, 8
Voltameter, 3
Voltmeter, 19, 104
——, calibration of, 35
——, electrostatic, 71, 74
——, principle of, 20
WATER analogy, 7
—— analogy of condenser, 42
Watt, 9
Wave form, 53
Wehnelt cathode, 186
Weston cell, 164
Wheatstone bridge, 21
Winding, bifilar, 130
Work, 8

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